NISTIR 4807

ON LOGARITHMIC RETINAE

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ABSTRACT

This paper suggests several iconic image ''warpings'', or remappings, which facilitate computationally inexpensive measurements of moving 3-D points relative to a camera. Assuming translational motion of the camera, where the optical axis coincides with the direction of motion, and a stationary scene, points in 3-D space that lie on a particular 3-D surface produce a constant value for some nonlinear function of the optical flow. This function need not be computed after the image is formed, but rather can be implemented by hardware at the retinal level, i.e., via non-linear variable-resolution (usually logarithmic) retina. Four sets of different surfaces are introduced and there is one optical-flow-based constant value for each surface. We call these values "invariants". An invariant, which is a scalar, describes a 3-D surface. For each invariant a logarithmic retina is defined which will cause optical flow on these surfaces to have identical values.

The process of image remapping, called ''normalization'', is defined for four 1-D parameterizations of space: range, depth, looming and clearance. For each invariant a camera-retina imaging model utilizing spherical projection and foveal peripheral resolution is described for analyzing optical flow. Computer simulation demonstrates how the new suggested retinae normalize the optical flow with respect to each one of the parameterizations.

This research was supported in part by a grant from the National Science Foundation to Florida Atlantic University (Grant #IRI-9115939).

1. INTRODUCTION

This paper describes nonlinear retinal mappings that simplify motion measurements of points in 3-D space. Assuming translational motion of a camera whose optical axis coincides with the direction of motion and a stationary scene, points in 3-D space that lie on a particular surface produce a constant value of some nonlinear function of the image optical flow. Four sets of surfaces are introduced, each of which corresponds to one optical-flow-based constant value (invariant). This means that the nonlinear function associated with a particular surface will result in the same value of optical flow function for all points on that surface, independent of where the points are in the image.

The nonlinear functions of optical flow can be implemented using multi-resolution (usually logarithmic) foveal-peripheral retinae and hence no computations are required. We list the four different surfaces that correspond to the 1-D parameterizations of 3-D space:

(1) *Constant Range Surface:* This surface corresponds to points in 3-D that lie at a constant range from the camera pinhole point, i.e., points lying on a sphere centered at the camera pinhole.

(2) *Constant Depth Surface:* This surface corresponds to points in 3-D which lie in the plane perpendicular to the optical axis of the camera, and are also referred to as constant *time to contact* points [LEE, RAVIV1].

(3) *Constant Looming Surface:* Looming of a point is defined [RAVIV2] as $-\dot{r}/r$ where r is the distance of the point from the observer and dot corresponds to the derivative with respect to time. It is a measure of an obstacle's collision threat. Points of constant looming lie on a sphere passing through the center of the camera pinhole point. The diameter of the sphere coincides with the translational motion vector of the camera.

(4) *Constant Clearance Surface:* This surface corresponds to points in 3-D which lie on a cylinder whose axis coincides with the translational vector i.e., points in 3-D which have a constant radial distance (or "clearance") from the camera [RAVIV1, ALBUS].

The analysis takes place in a spherical ($R-\Theta-\Phi$) coordinate system. In this representation, since we deal with translation only along the optical axis, the projection of any 3-D point in this coordinate system moves along a constant ϕ radial line and can be processed independently of any other point.

Similar ideas have been documented in a number of places, e.g., [WEIMAN, FISHER, MESSNER, SANDINI], for the "log-polar" transform. The log-polar remapping is the planar projection image "normalized for depth".

2. SPHERICAL COORDINATES AND PROJECTION FOR 3-D SPACE

In a rectilinear motion with no rotation, points in the image plane move away from the Focus of Expansion (FOE) (Figure 1a) and towards the Focus of Contraction (FOC) (Figure 1b). Based on this observation we use an R-Θ-Φ spherical rather than a Cartesian representation of points in space, which reduces to a Θ-Φ representation in the image domain.

Figures 2 and 3 show the chosen coordinate system and the definitions of r and the angles θ and ϕ. (Note that the R-Θ-Φ coordinate system corresponds to the velocity egosphere defined in [ALBUS].) If the optical axis coincides with the translational vector, then in the image domain, constant ϕ corresponds to a radial line that emerges from the FOE and a constant θ corresponds to a circle whose center is the FOE. Given a point in Cartesian coordinates, it can be transformed to a $(r-\theta-\phi)$ point in the $(R-\Theta-\Phi)$ domain and vice versa.

3. PROJECTIVE SURFACE INVARIANTS

This section summarizes previous results on the invariants [RAVIV1]. For a translational motion of the camera, for any point in space (except for $\theta=0$ and $\theta=\pi$) it has been shown that:

$$\frac{v}{r} = \frac{\dot{\theta}}{\sin\theta} \tag{1}$$

Here, v is the translational speed of the observer, and the dot denotes differentiation with respect to time (See Fig. 4). $\dot{\theta}$ is the optical flow along a radial line, i.e., for constant ϕ. At any point in time v is constant, and hence the value $r\frac{\dot{\theta}}{\sin\theta}$ is the same for *all points* in 3-D space, except those with $\theta=0$ and $\theta=\pi$. The values θ and $\dot{\theta}$ can be measured/computed for a point in stationary 3-D space at each instant of time, and hence the nonlinear function $\frac{\dot{\theta}}{\sin\theta}$ can be obtained.

The following is a description of the four invariants in terms of optical flow. The derivations hold for all points in 3-D except those that lie on the motion axis, i.e., with $\theta=0$ and $\theta=\pi$. We denote the invariants by $\frac{1}{\tau_R}$, $\frac{1}{\tau_S}$, $\frac{1}{\tau_C}$, and $\frac{1}{\tau_P}$. They all have units of $\frac{1}{time}$. Figure 5 is a summary of the invariants. It shows the basic relationship between space, speed and optical flow (top equation), from which the four invariants are derived. Based on geometrical properties, the time-dependent invariants are shown as a function of optical flow. The geometrical interpretations of all invariants as 1-dimensional parameterizations are shown at the bottom of Figure 5.

1. *The constant range invariant* $\frac{1}{\tau_R}=\frac{\dot{\theta}}{\sin\theta}$. All points in 3-D space that lie on a sphere whose center is the pinhole point of the camera share this invariant, i.e., have the same τ_R. The meaning of this invariant is that the modified optical flow $\frac{\dot{\theta}}{\sin\theta}$ is the *same* for all points on a sphere (except those which are on the axis motion of the camera). Points inside the sphere ("close" points) produce higher values of $\frac{\dot{\theta}}{\sin\theta}$ and points outside the sphere ("far" points) produce smaller values of $\frac{\dot{\theta}}{\sin\theta}$. Hence, it is possible to find to within a scale factor the range of a point by simply calculating this value.

2. *The constant looming invariant* $\frac{1}{\tau_S}=\frac{\dot{\theta}}{\tan\theta}$. All points in 3-D space that lie on a sphere which lies in front of the camera share this invariant, i.e., have the same τ_S. The sphere diameter coincides with the optical axis of the camera, and the camera lies on the sphere's surface. In this case the diameter of the sphere $\frac{r}{\cos\theta}$ is constant, and so, using Equation (1), $\frac{\dot{\theta}}{\tan\theta}$ remains constant. It has been shown by [RAVIV2] that all points on a particular sphere result in the same visual "looming".

3. *The constant clearance invariant* $\frac{1}{\tau_C}=\frac{\dot{\theta}}{\sin^2\theta}$. All points in 3-D space that lie on a cylindrical surface whose axis coincides with the camera translational motion vector share this invariant, i.e., have the same τ_C.

4. *The constant depth (time to contact) invariant* $\frac{1}{\tau_P}=\frac{2\dot{\theta}}{\sin 2\theta}$. All points in 3-D space that lie on a plane which is perpendicular to the direction of motion of the camera, share this invariant, i.e., have the same τ_P. τ_P is the "time to contact" as described by [LEE]. It tells the animal how to control its motion in order to avoid collision.

4. VARIABLE RESOLUTION RETINAE

In this section we show how these invariants may be exploited by an appropriate "retina" to normalize optical flow. For purposes here, it is immaterial whether this retina is implemented optically, as spatially variant receptor arrays, or by explicit non-linear subpixel sampling.

[ORSER] gives additional motivation as well as describing a binocular wire frame scene simulator which gives simulated results for all four retinae.

For each invariant we show the corresponding structure of the retina. Range, depth, looming and clearance normalizations correspond to the previously described range, depth, looming and clearance invariants, respectively.

4.1. RANGE NORMALIZATION

Equation (1) describes the basic relationship between range, velocity and optical flow for a forward translating camera. First we rewrite it as:

$$\frac{d\theta}{\sin\theta} = \frac{v}{r}dt, \tag{2}$$

Another way of rewriting Equation (1) is

$$\frac{d}{dt}\ln(tan\frac{\theta}{2}) = \frac{v}{r} \tag{3}$$

When integrating both sides of Equation (2) (for fixed r and v) we obtain:

$$\ln\tan\frac{\theta}{2} - \ln\tan\frac{\theta_0}{2} = \frac{v(t-t_0)}{r} \tag{4}$$

The meaning of Equations (3) and (4) is that a $ln(tan\frac{\theta}{2})$ retina will produce the same optical flow for all points that lie at the same distance from the observer, i.e., $\frac{\dot{\theta}}{\sin\theta}$ will be measured linearly. For the $ln(tan\frac{\theta}{2})$ retina, the optical flow magnitude is in a simple inverse relationship to range.

We call this remapping *range normalization,* since the magnitude of optical flow values will be equal if and only if they are generated by points having the same range.

Figure 6a shows 3-D points which are at the same range from the moving observer. The optical flow $\dot{\theta}$ generated in the image plane by these points (for a forward translating camera) is shown in Figure 6b. Note that the optical flow values vary from one point to another as a function of radial angle θ. However, the range-normalized optical flow for these points $\frac{\dot{\theta}}{\sin\theta}$ is identical for all the points (Figure 6c) and is generated directly by a $\ln(\tan\frac{\theta}{2})$ retina.

If ϕ and θ are plotted against each other, the resulting representation of the sphere is called the isometric plane. Figure 6d depicts the optical flow in this manner while figure 6e is the range-normalized optical flow using an analogous logarithmic isometric plane representation.

The nonlinear radial displacement as a function of retinal eccentricity and resolution elements per unit of eccentricity for the range normalizing retina are shown as the solid and dashed lines of Figure 6f, respectively.

4.2. DEPTH NORMALIZATION

Instead of keeping range r constant, as in the preceeding section, it may be desirable to keep depth (i.e., the projection of range onto the optical axis) constant. Points at a common depth X from the camera-retina are parameterized by θ and in terms of range r are given by :

$$X = r \cos\theta \qquad 0 \le \theta \le 90°. \tag{5}$$

Hence by dividing Equation (1) by cos θ, we get:

$$\frac{v}{r\cos\theta} = \frac{1}{\cos\theta}\frac{\dot\theta}{\sin\theta} \tag{6}$$

or:

$$\frac{v}{r\cos\theta}dt = \frac{2}{\sin 2\theta}d\theta \tag{7}$$

Another way of writing Equation (6) is:

$$\frac{d}{dt}\ln(\tan\theta) = \frac{v}{r\cos\theta} \tag{8}$$

After integrating both sides of Equation (7) we obtain:

$$\ln\tan\theta - \ln\tan\theta_0 = \frac{v}{r\cos\theta}(t - t_0) \tag{9}$$

The meaning of Equations (8) and (9) is that a $\ln(\tan\theta)$ retina will produce the same optical flow for all points that lie at the same depth $\frac{v}{r\cos\theta}$ from the observer, i.e., $\frac{\dot\theta}{\sin\theta\cos\theta}$ (which equals $\frac{2\dot\theta}{\sin 2\theta}$) will be measured linearly.

For the $\ln(\tan\theta)$ retina, the optical flow is in a simple inverse relationship to depth.

We call this remapping *depth normalization,* since the magnitude of optical flow values will be equal if and only if they are generated by points having the same depth.

Figure 7a shows 3-D points which lie in the same depth from the moving observer. The optical flow $\dot{\theta}$ generated by these points is shown in Figure 7b. The normalized optical flow for points in 3-D with the same range $\frac{2\dot{\theta}}{\sin 2\theta}$ is identical for all the points (Figure 7c). This value is measured directly by a $ln\,(tan\,\theta)$ retina.

The isometric plane and logarithmic isometric plane representations are shown in Figures 7d and 7e, respectively.

The nonlinear radial displacement as a function of retinal eccentricity and resolution elements per unit of eccentricity for the depth normalizing retina are shown as the solid and dashed lines of Figure 7f, respectively.

This depth normalization is the spherical projection analog of the log-polar transform [WEIMAN, FISHER].

4.3. LOOMING NORMALIZATION

Spheres of constant looming have been discussed in [RAVIV2] and refer to points at range r lying on a sphere whose diameter is given by $r / \cos\theta$. *Looming normalization* refers to the mapping of the spherical projection in such a way that optical flow values of two points are equal if and only if they lie on the same sphere of constant looming.

These points of the sphere can be parameterized by the reciprocal of the radius of the sphere in terms of θ and range r as:

$$\frac{1}{radius\ \ of\ \ sphere} = \frac{\cos\theta}{r}, \qquad 0 \le \theta \le 90^\circ. \tag{10}$$

Hence by multiplying Equation (1) by $\cos\theta$, we get:

$$\frac{v\ \cos\theta}{r} = \cos\theta \frac{\dot{\theta}}{\sin\theta} \tag{11}$$

or

$$\frac{v\ \cos\theta}{r} dt = \frac{d\theta}{\tan\theta} \tag{12}$$

Equation (11) can also be rewritten as:

$$\frac{d}{dt}\ln(\sin\theta) = \frac{v\ \cos\theta}{r} \tag{13}$$

After integrating both sides of Equation (12) we obtain:

$$\ln \sin \theta - \ln \sin \theta_0 = \frac{v \cos \theta}{r}(t - t_0) \tag{14}$$

The meaning of Equations (13) and (14) is that a ln(sinθ) retina will produce the same optical flow for all points that lie on the same looming sphere, i.e., $\frac{\dot{\theta}}{\tan\theta}$ will be measured linearly.

For the $ln(sin\,\theta)$ retina, the optical flow produced by it has a simple direct relationship to the looming sphere on which it lies.

We call this remapping *looming normalization,* since the magnitude of optical flow values will be equal if and only if they are generated by points having the same looming value.

Figure 8a shows 3-D points which lie in the same looming sphere. The optical flow $\dot{\theta}$ generated by these points is shown in Figure 8b. The normalized optical flow for these points, $\frac{\dot{\theta}}{\tan\theta}$, is identical for all the points (Figure 8c). This value is measured directly by a $ln(sin\,\theta)$ retina.

The isometric plane and logarithmic isometric plane are shown in Figures 8d and 8e.

The nonlinear radial displacement as a function of retinal eccentricity and resolution elements per unit of eccentricity for the looming normalizing retina are shown as the solid and dashed lines of Figure 8f, respectively.

4.4. CLEARANCE NORMALIZATION

In the situation in which the camera-retina is translating along a straight line, the prediction of obstacles adjacent to this line which it will not "clear" is desirable. The locus of these points for a constant radius of lateral clearance is a cylinder whose axis is collinear with the axis of translation. [ALBUS, RAVIV1]

A point at range r will lie on the cylinder whose radius is given by $r \sin \theta$. Hence $\frac{v}{r \sin \theta}$ is constant for *all* points on the cylinder, and so by dividing Equation (1) by sinθ we have:

$$\frac{v}{r \sin \theta} = \frac{1}{\sin \theta} \frac{\dot{\theta}}{\sin \theta} \tag{15}$$

or

$$\frac{v}{r \sin \theta} dt = \frac{1}{\sin^2 \theta} d\theta \tag{16}$$

Another way of writing Equation (15) is:

$$-\frac{d}{dt}\cot\theta = \frac{v}{r\sin\theta} \tag{17}$$

By integrating both sides of Equation (16) we obtain:

$$\frac{v}{r\sin\theta}(t - t_0) = -(\cot\theta - \cot\theta_0) \tag{18}$$

The meaning of Equations (17) and (18) is that a $\cot\theta$ retina will produce the same optical flow for all points that lie at the same radial distance from the moving observer, i.e., $\frac{\dot{\theta}}{\sin^2\theta}$ will be measured linearly.

Note that this retina is the only one which is not logarithmic. However, it can be written as a logarithmic one by using the identity:

$$\cot\theta = \ln e^{\cot\theta} \tag{19}$$

The optical flow produced by the $\cot\theta$ retina is directly proportional to the inverse of the radius of the cylinder, i.e., the clearance the object has with respect to the trajectory of the camera.

We call this remapping *clearance normalization,* since the magnitude of optical flow values will be equal if and only if they are generated by points having the same clearance (radial distance).

Figure 9a shows 3-D points which lie in the same cylinder surrounding the observer (each point on the cylinder has the same distance from the translation axis). The optical flow $\dot{\theta}$ generated by these points is shown in Figure 9b. Note that the optical flow values vary from one point to another. However, the clearance normalized optical flow for these points $\frac{\dot{\theta}}{\sin^2\theta}$ is identical for all the points (Figure 9c). This value is measured directly by a $\cot\theta$ retina.

The isometric plane and logarithmic isometric plane representations for the unnormalized and clearance normalized optical flow is shown in figures 9d and 9e respectively.

The nonlinear radial displacement as a function of retinal eccentricity and resolution elements per unit of eccentricity for the clearance normalizing retina are shown as the solid and dashed lines of Figure 9f, respectively.

5. CONCLUSIONS

In this paper we have shown several retinal mappings. They are extensions of the well known log-polar retina as described by [WEIMAN, SANDINI] and others. Each retina described in this paper allows simple and direct measurements of a function of optical flow. A value of this function corresponds to a 3-D surface.

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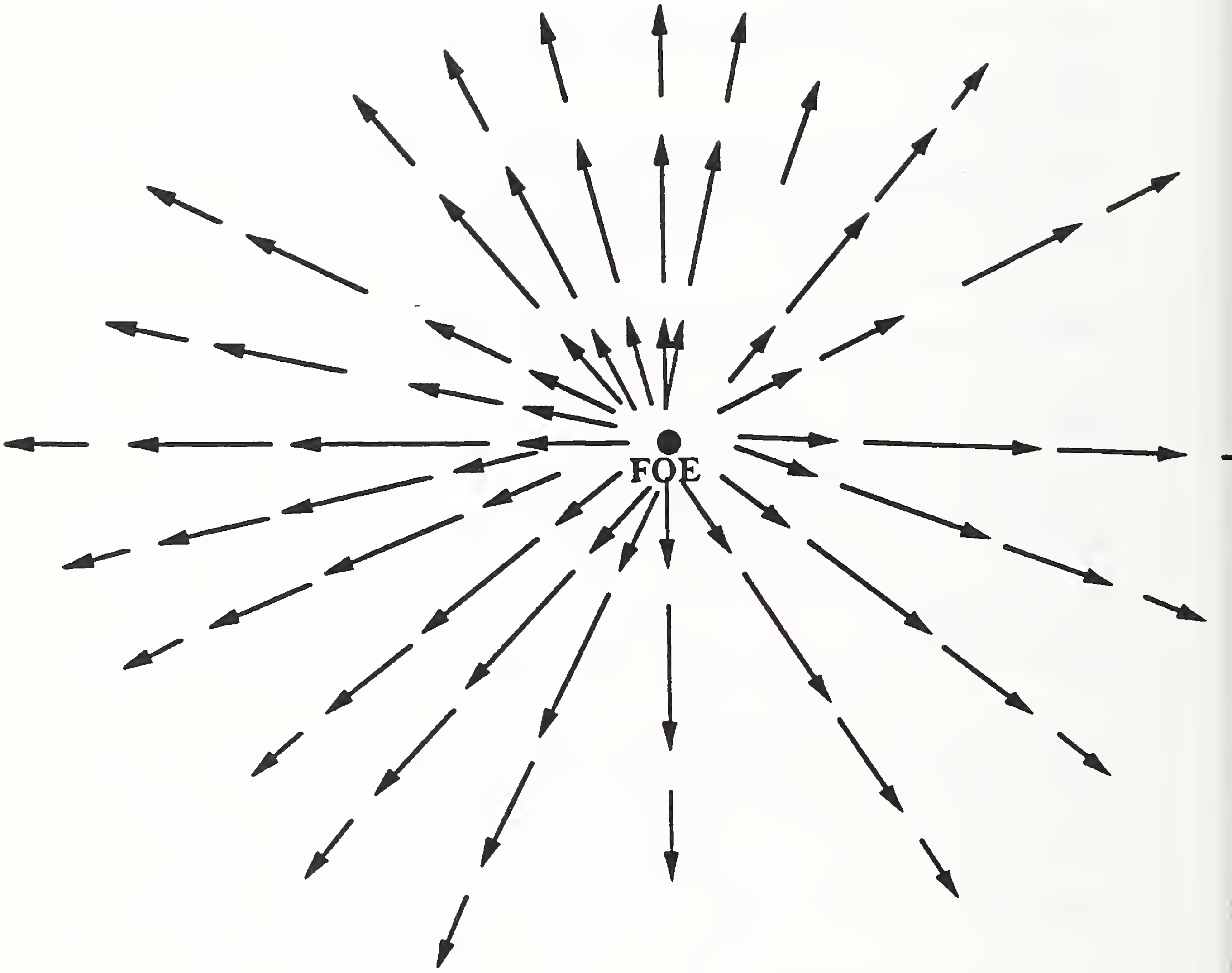


Figure 1a: OPTICAL FLOW RELATIVE TO THE FOCUS O
EXPANSION (FOE)

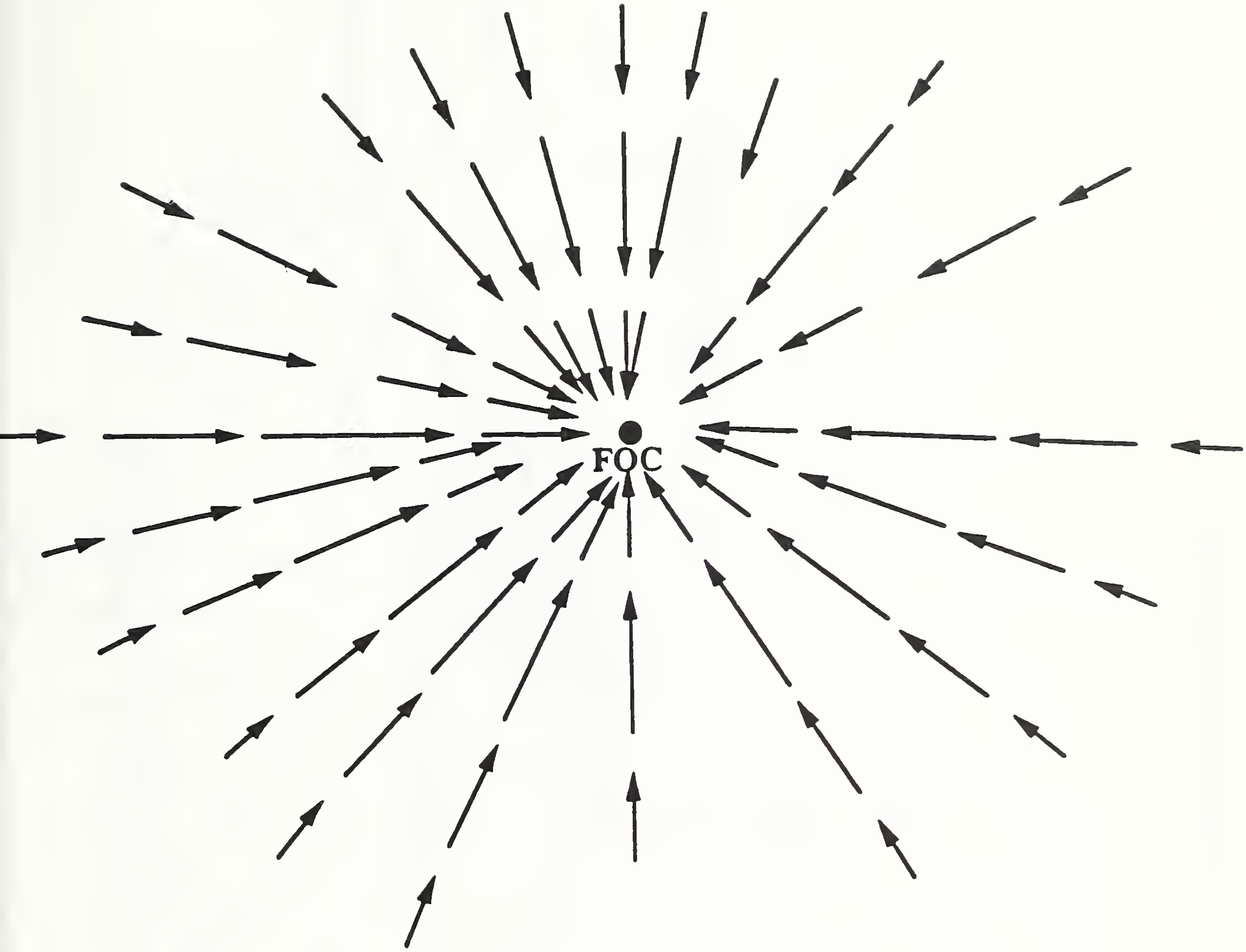


Figure 1b: OPTICAL FLOW RELATIVE TO THE FOCUS OF CONTRACTION (FOC)

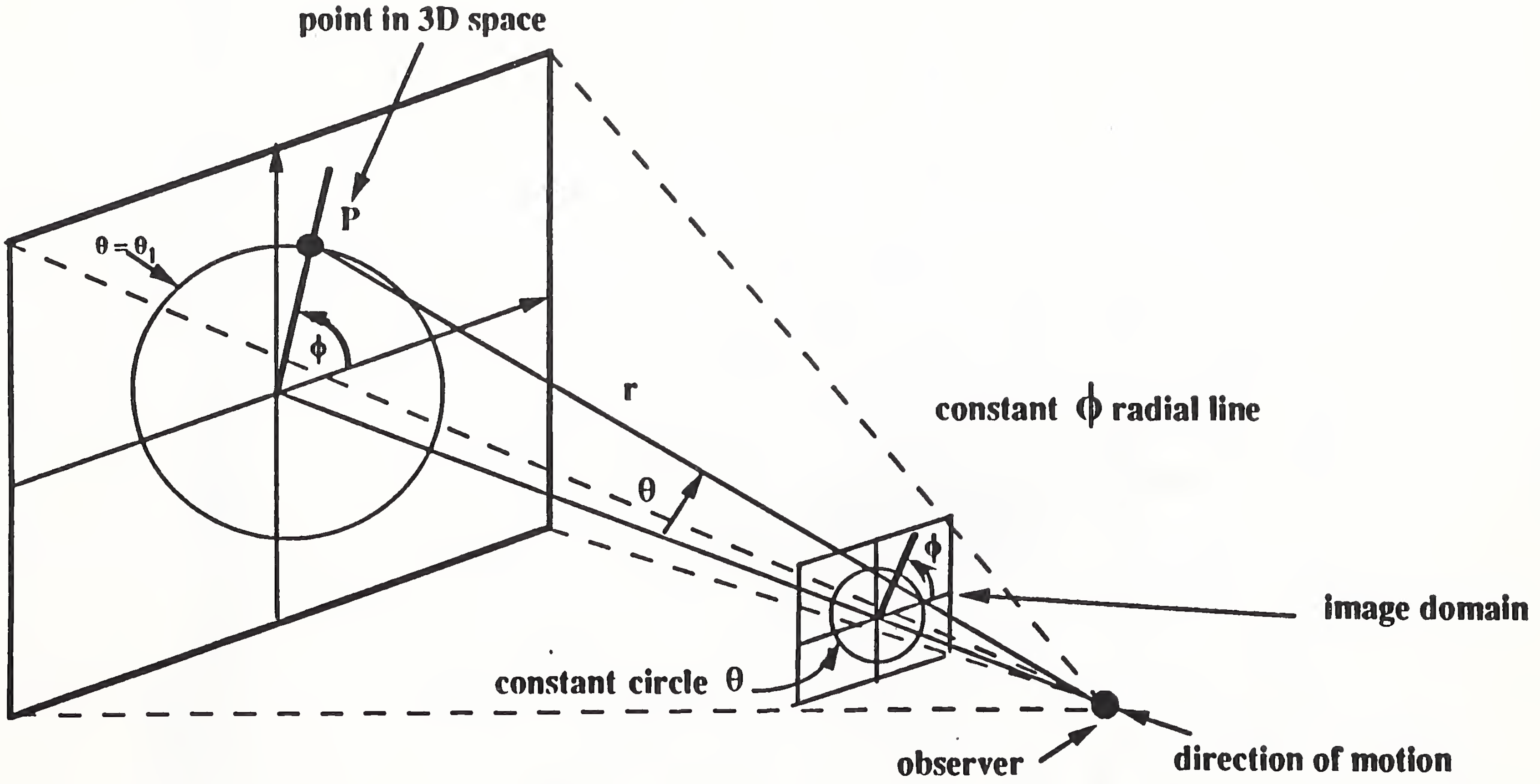


Figure 2: R–Θ–Φ COORDINATE SYSTEM

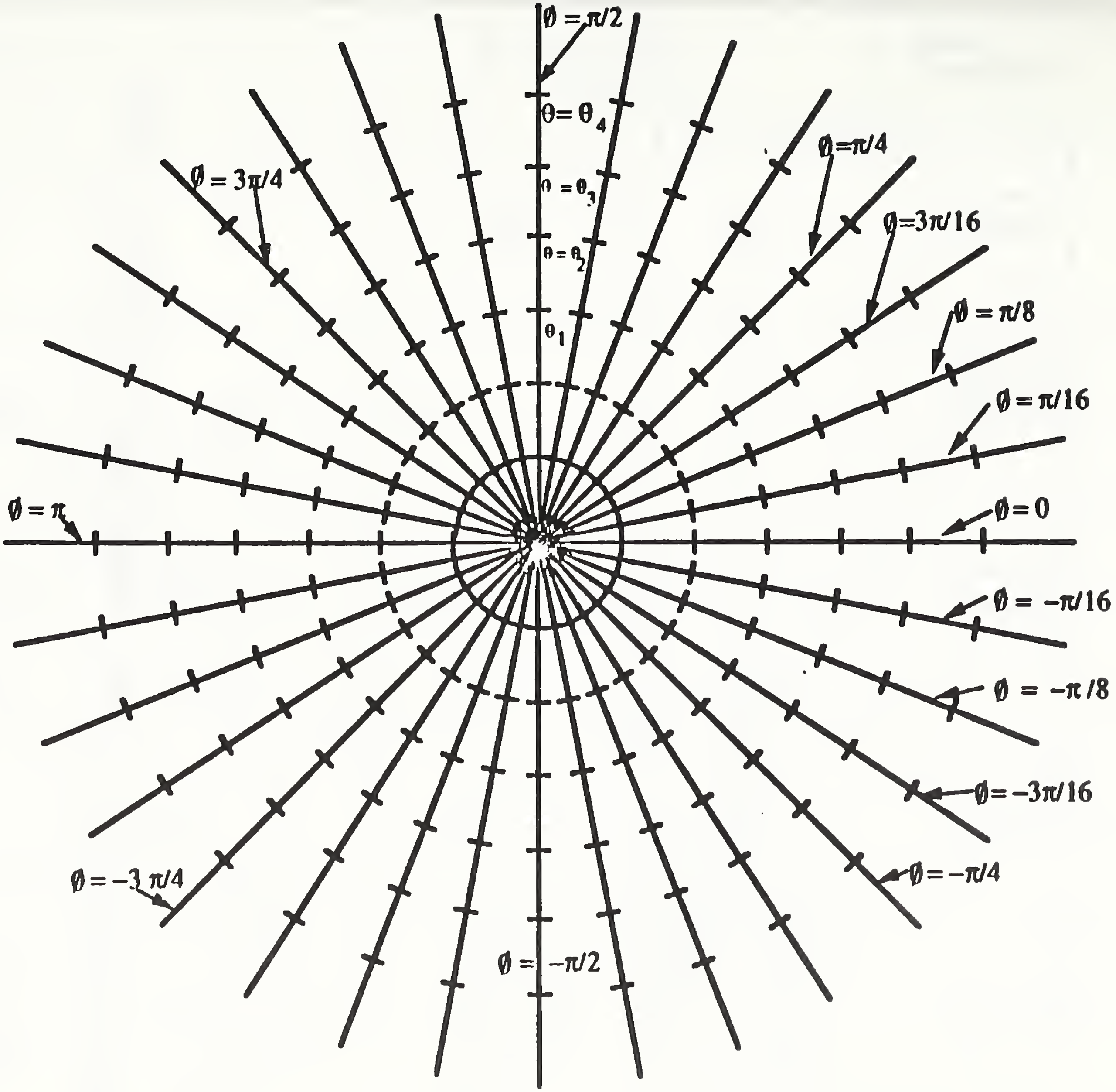


Figure 3: THE Θ–Φ IMAGE DOMAIN

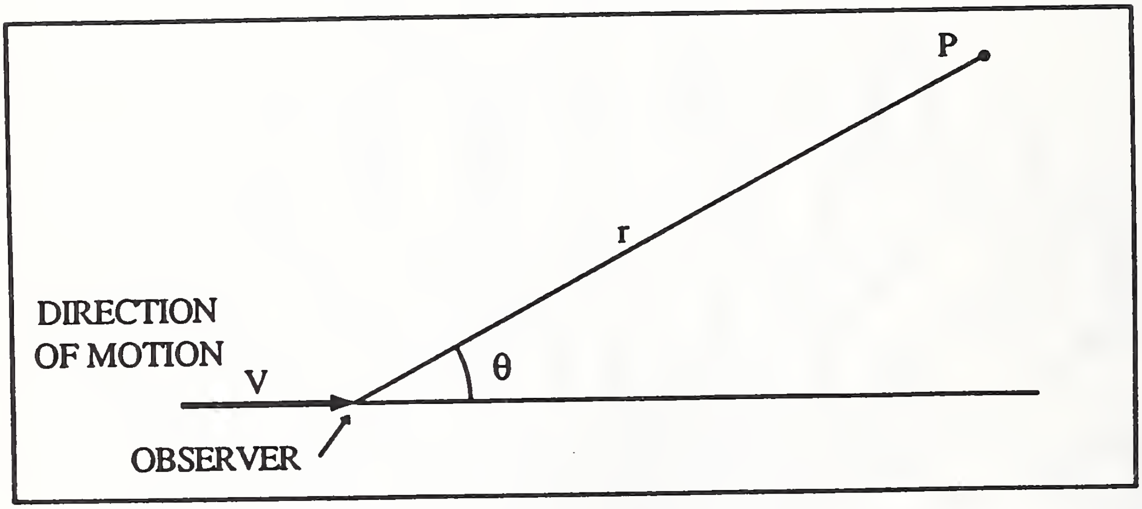


Figure 4: GEOMETRY FOR CONSTANT Φ RADIAL LINE

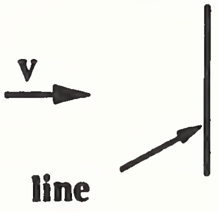
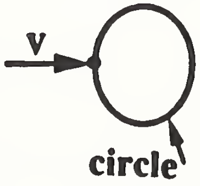
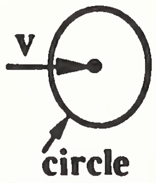
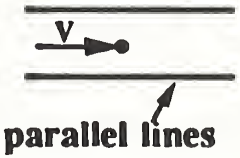
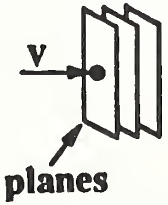
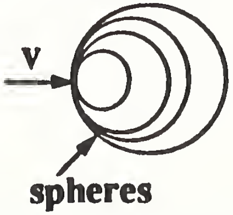
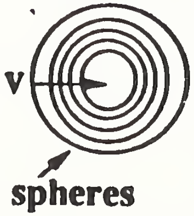
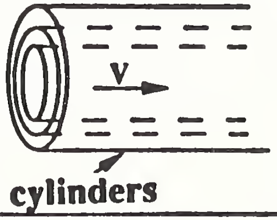
$V = r \dfrac{\dot{\theta}}{\sin\theta}$				**Basic Relationship**
$\dfrac{1}{T_P}$	$\dfrac{1}{T_S}$	$\dfrac{1}{T_R}$	$\dfrac{1}{T_C}$	**notation of invariant**
$r \cos\theta$	$\dfrac{r}{\cos\theta}$	r	$r \sin\theta$	**geometrical property of invariance**
$\left(\dfrac{\dot{\theta}}{\sin\theta}\right) \dfrac{1}{\cos\theta} = \dfrac{2\dot{\theta}}{\sin 2\theta}$	$\left(\dfrac{\dot{\theta}}{\sin\theta}\right) \cos\theta = \dfrac{\dot{\theta}}{\tan\theta}$	$\left(\dfrac{\dot{\theta}}{\sin\theta}\right) 1 = \dfrac{\dot{\theta}}{\sin\theta}$	$\left(\dfrac{\dot{\theta}}{\sin\theta}\right) \dfrac{1}{\sin\theta} = \dfrac{\dot{\theta}}{\sin^2\theta}$	**invariant in terms of optical flow**
				**geometrical interpretation in 2-D**
				**geometrical interpretation in 3-D**

Figure 5: SUMMARY OF THE INVARIANTS

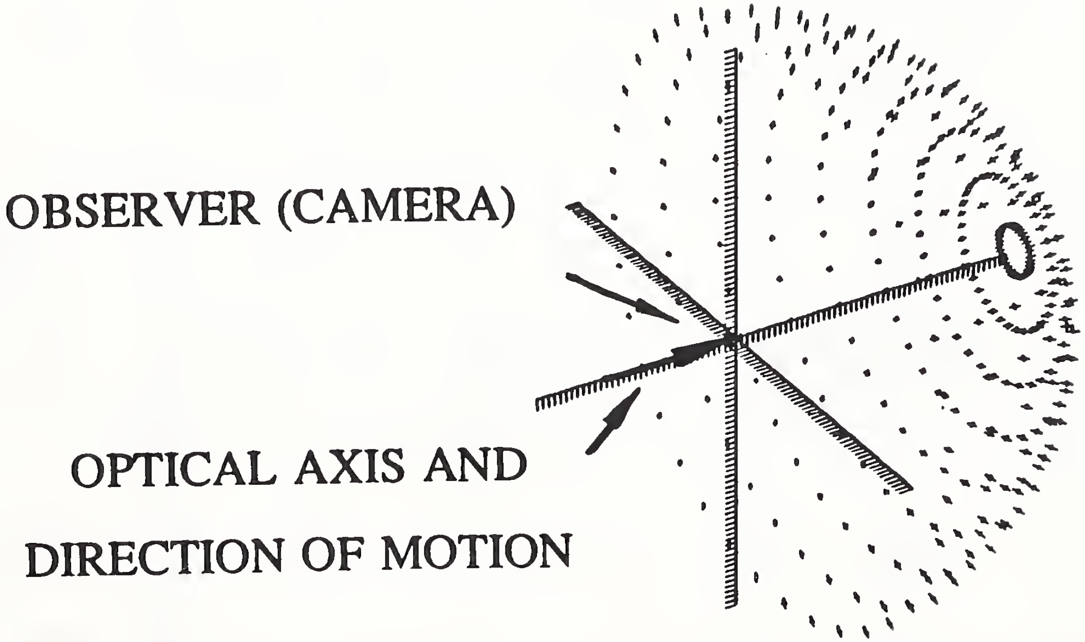


Figure 6a: EQUAL RANGE POINTS

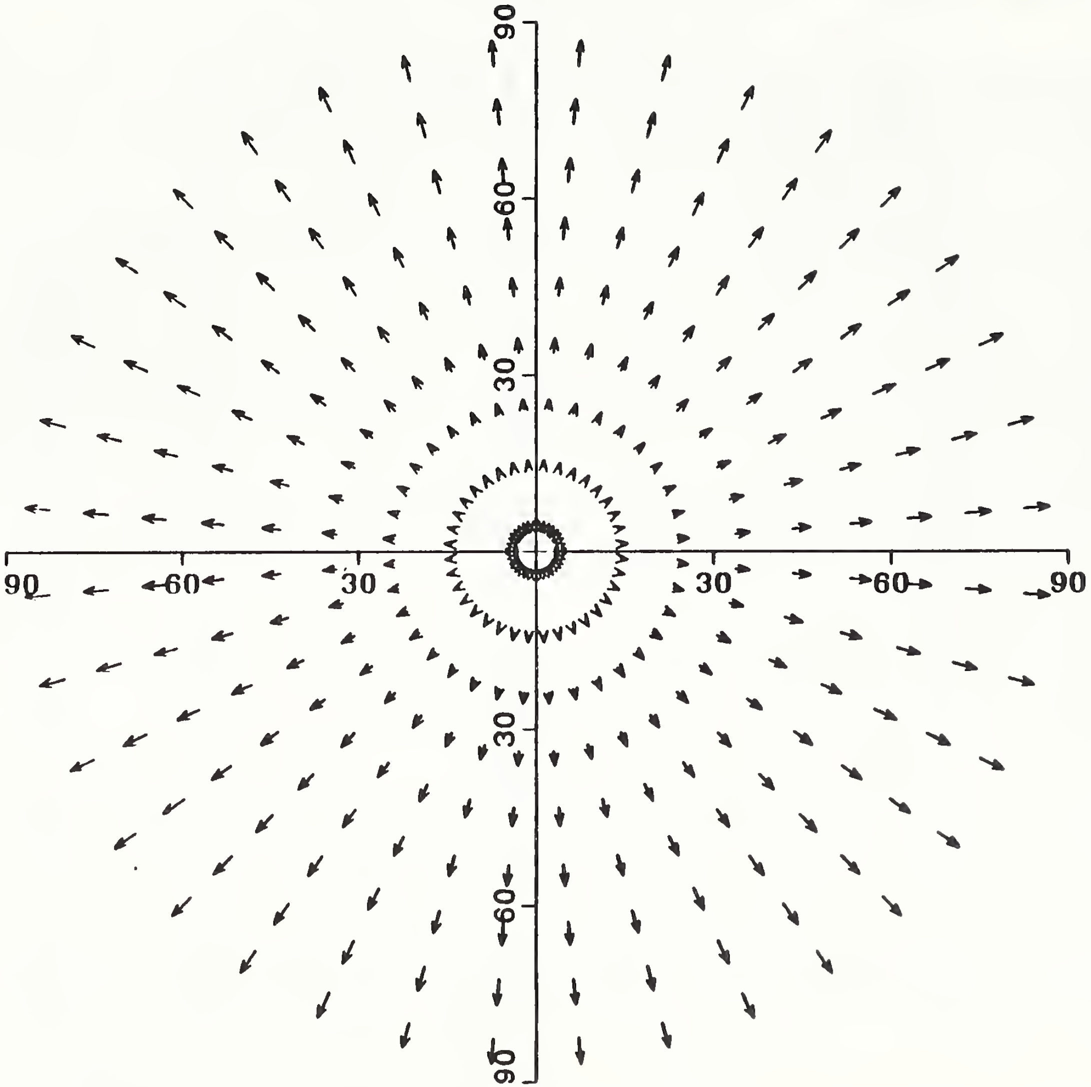


Figure 6b: OPTICAL FLOW GENERATED BY POINTS WITH CONSTANT RANGE

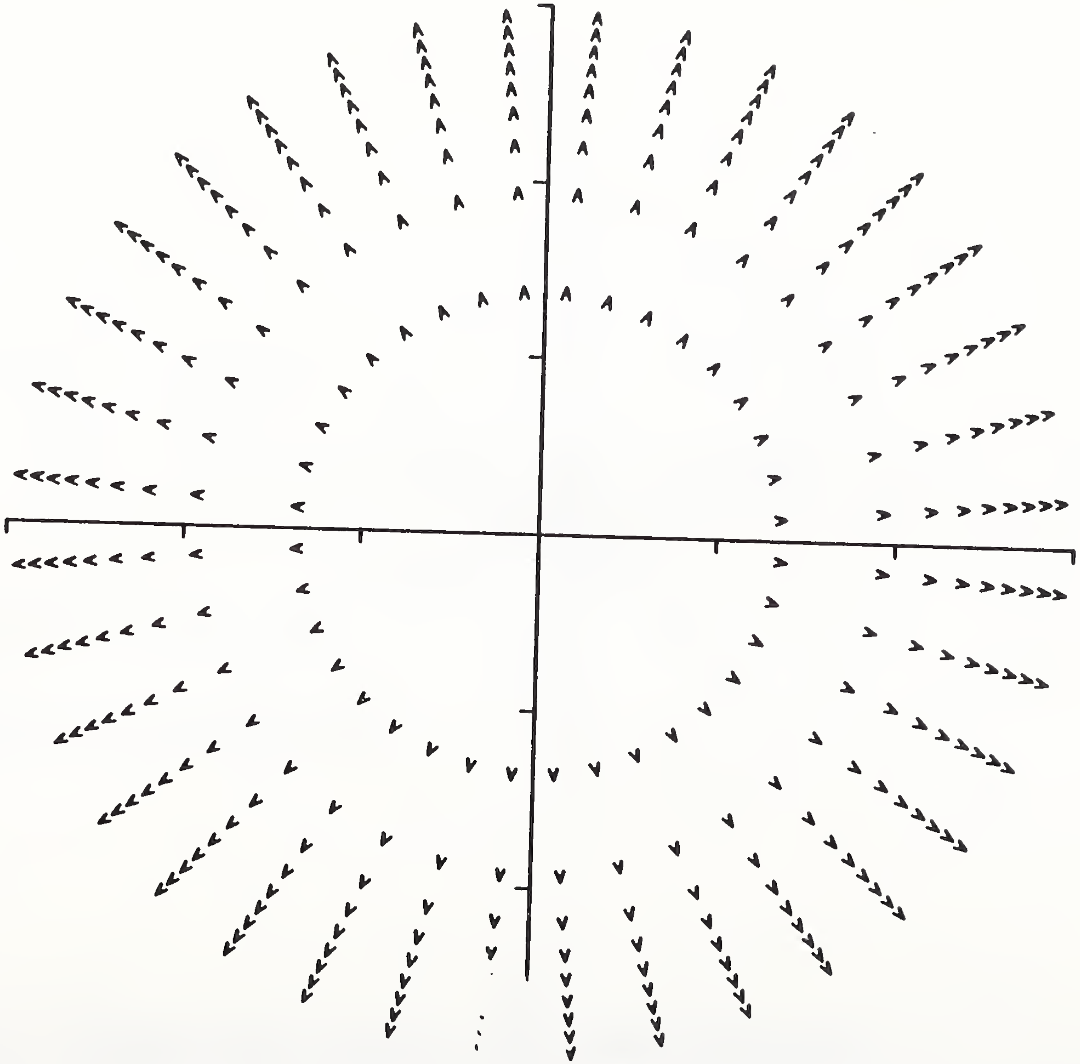


Figure 6c: RANGE NORMALIZATION OF OPTICAL FLOW

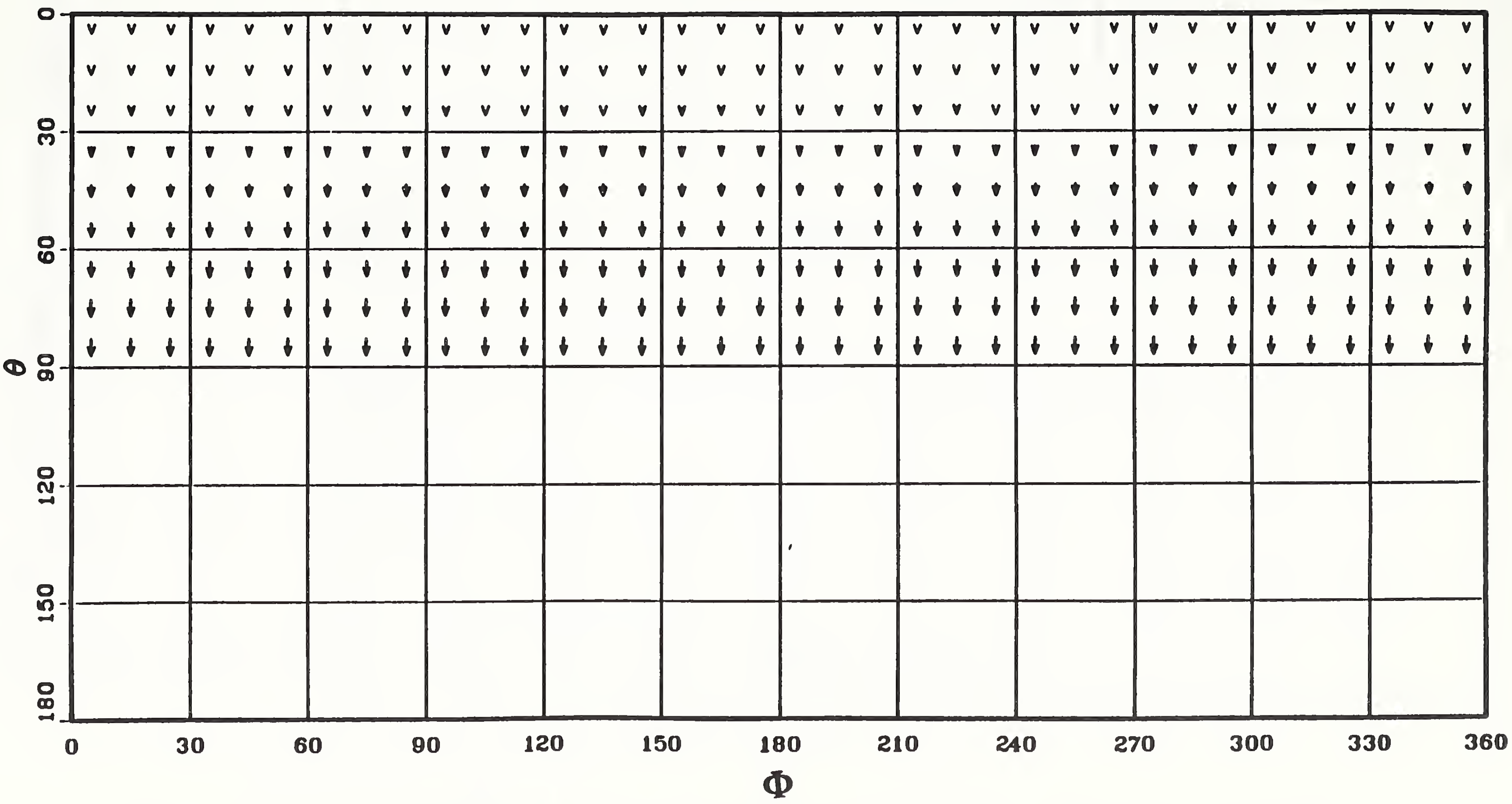


Figure 6d: ISOMETRIC REPRESENTATION OF OPTICAL FLOW

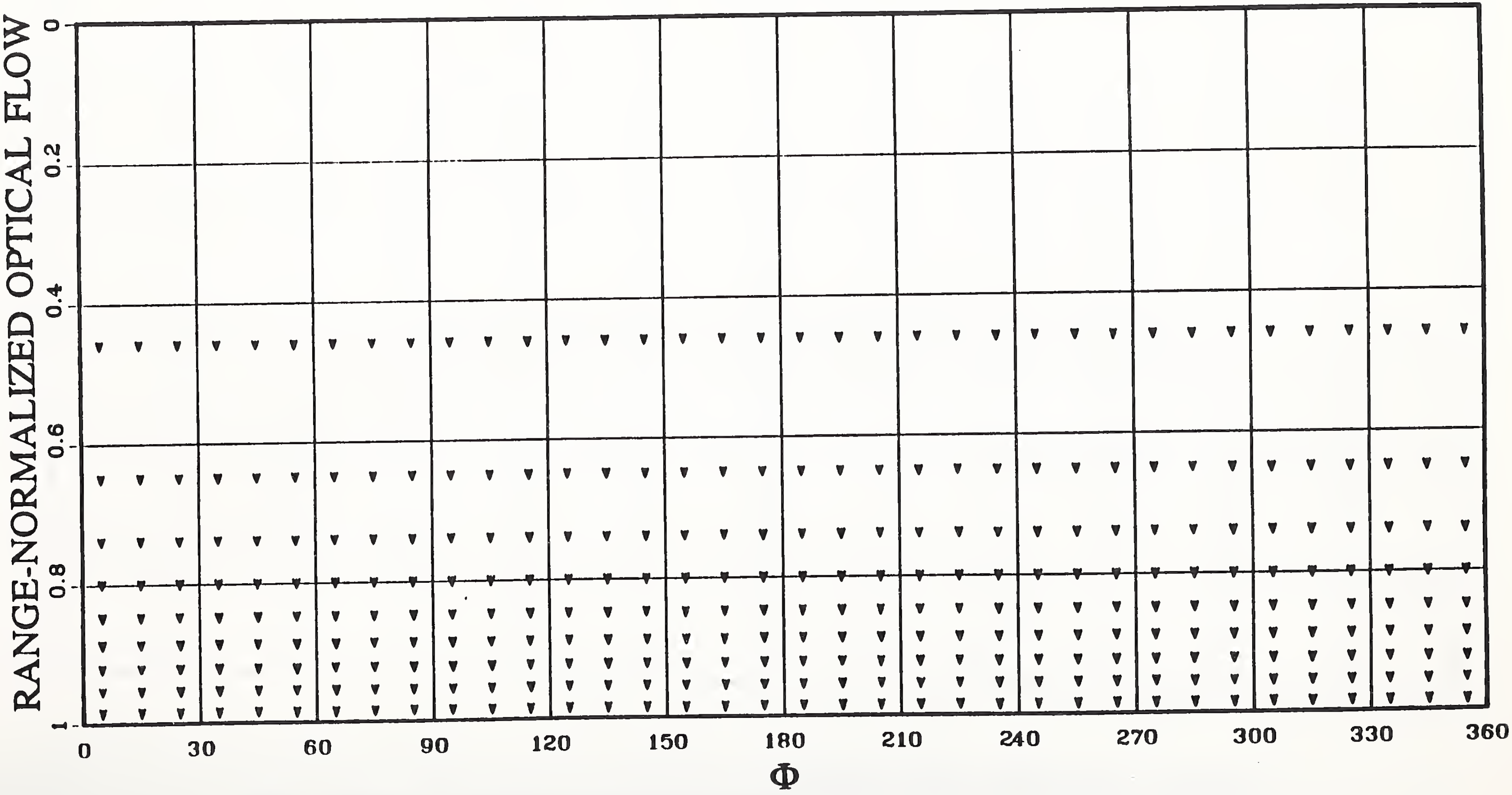
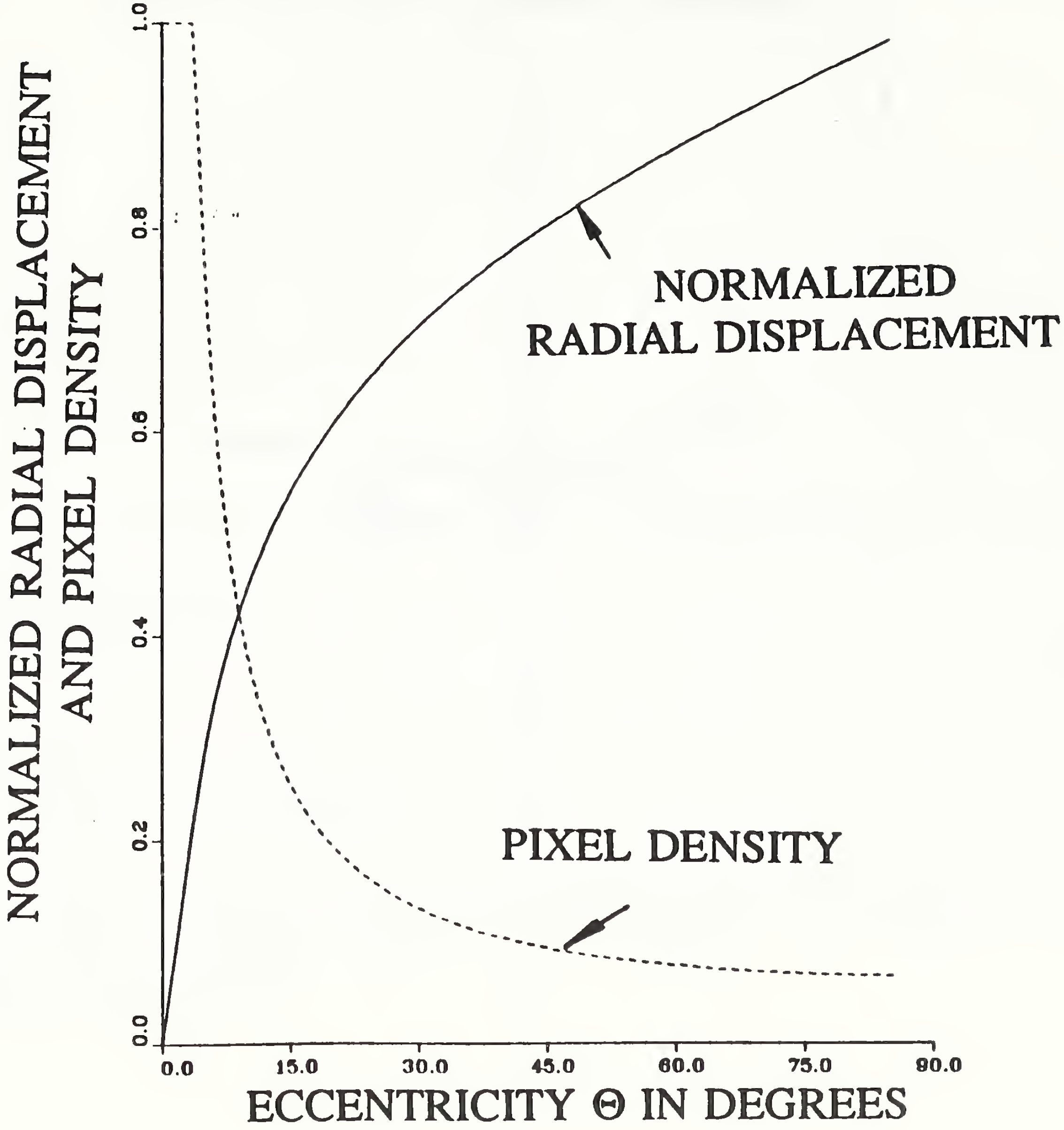


Figure 6e: ISOMETRIC REPRESENTATION OF RANGE-NORMALIZED OPTICAL FLOW

Figure 6f: LOG TAN $\frac{\Theta}{2}$ RETINA (RANGE)



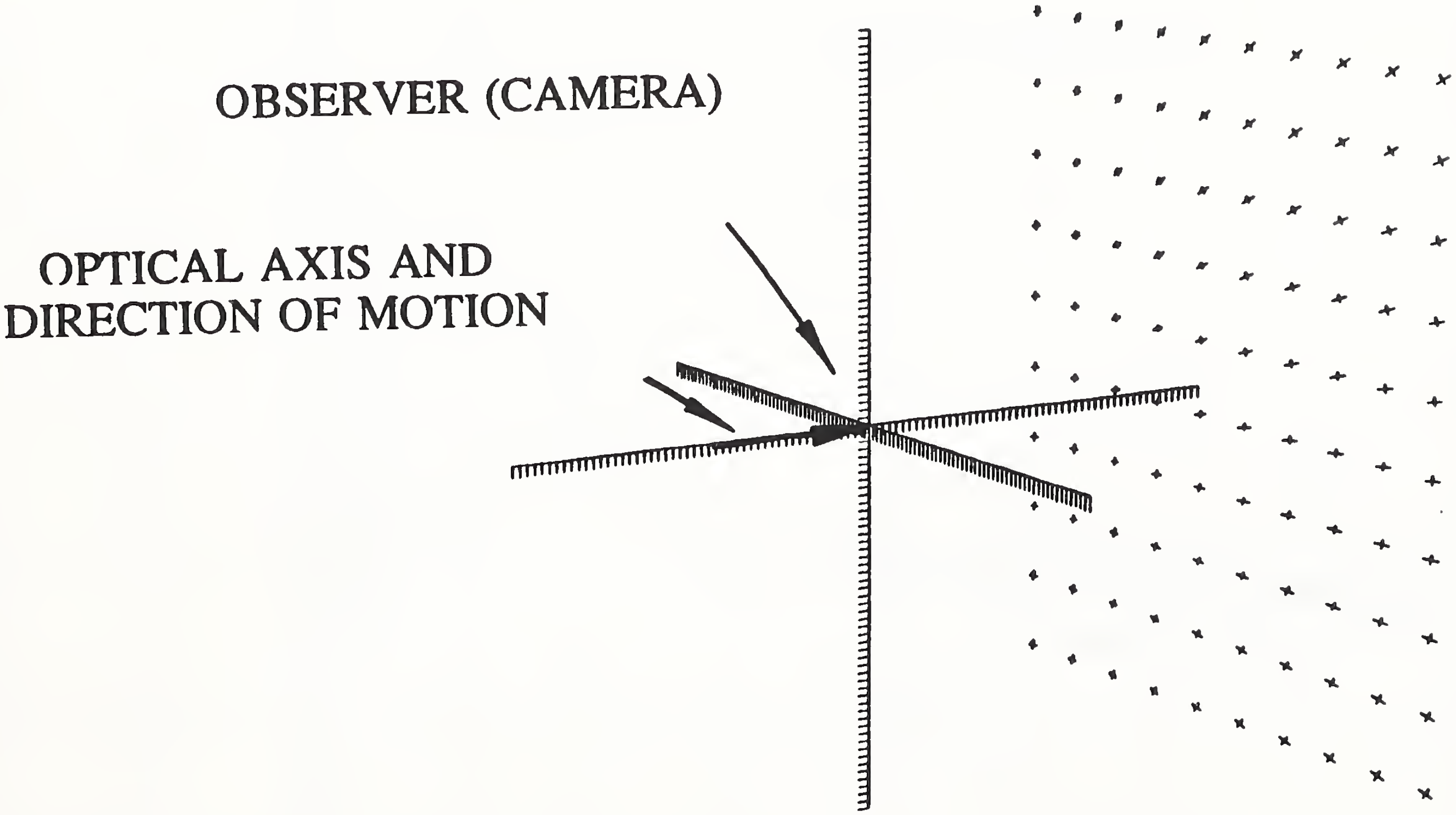


Figure 7a: EQUAL DEPTH POINTS

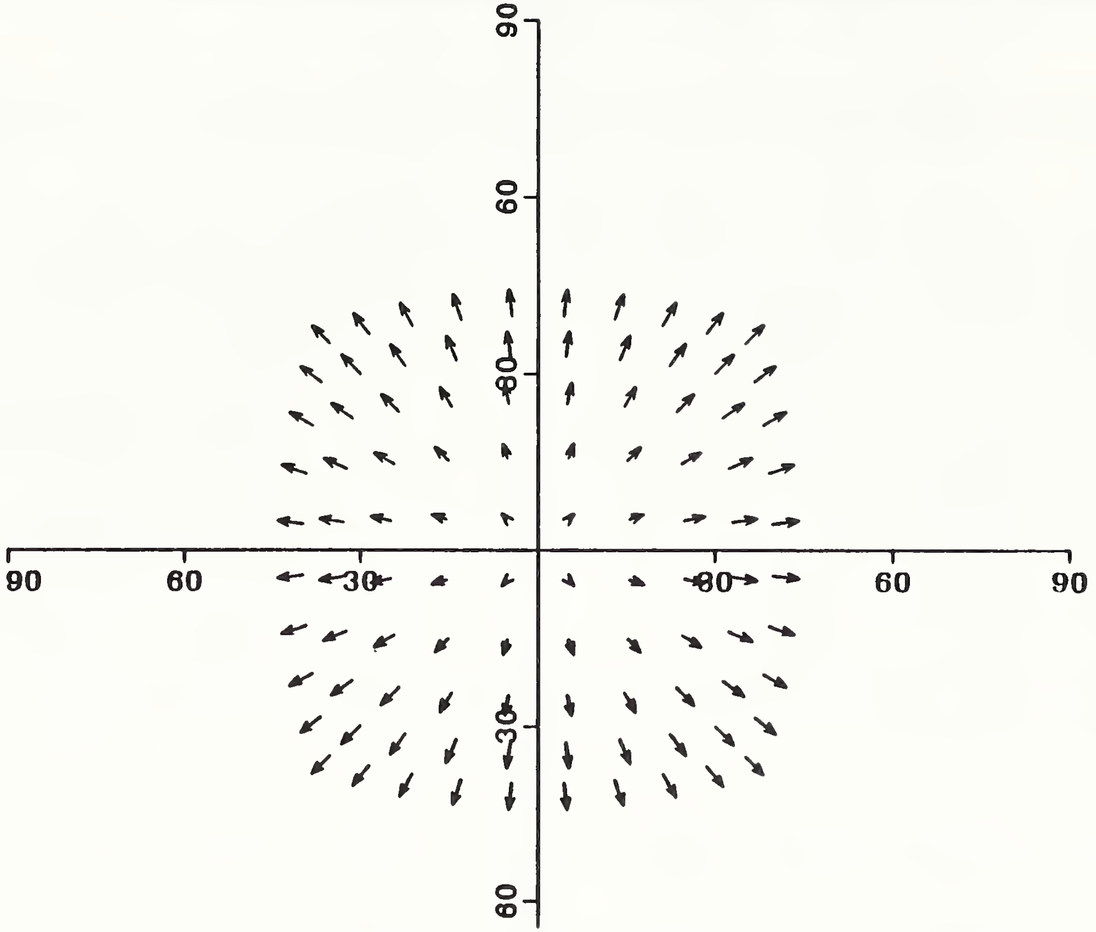


Figure 7b: OPTICAL FLOW GENERATED BY POINTS WITH CONSTANT DEPTH

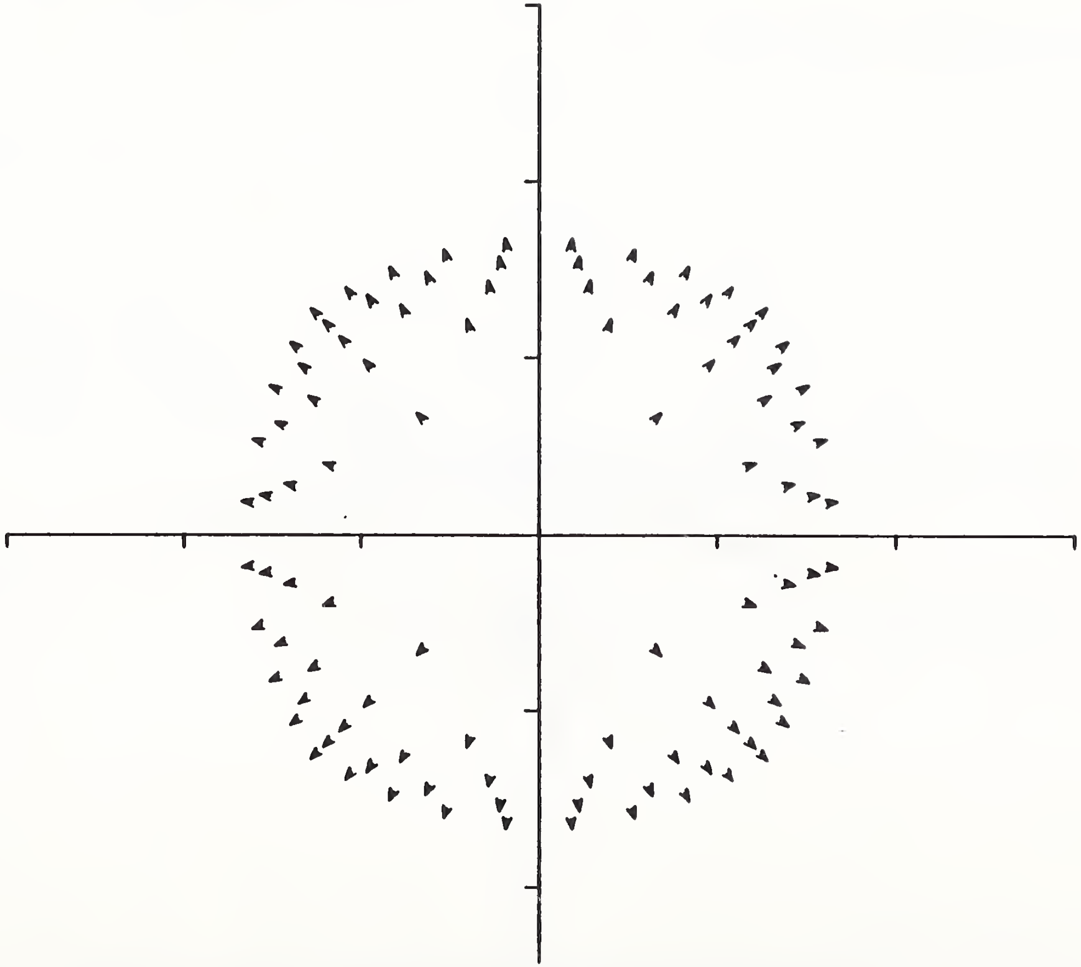


Figure 7c: DEPTH NORMALIZATION OF OPTICAL FLOW

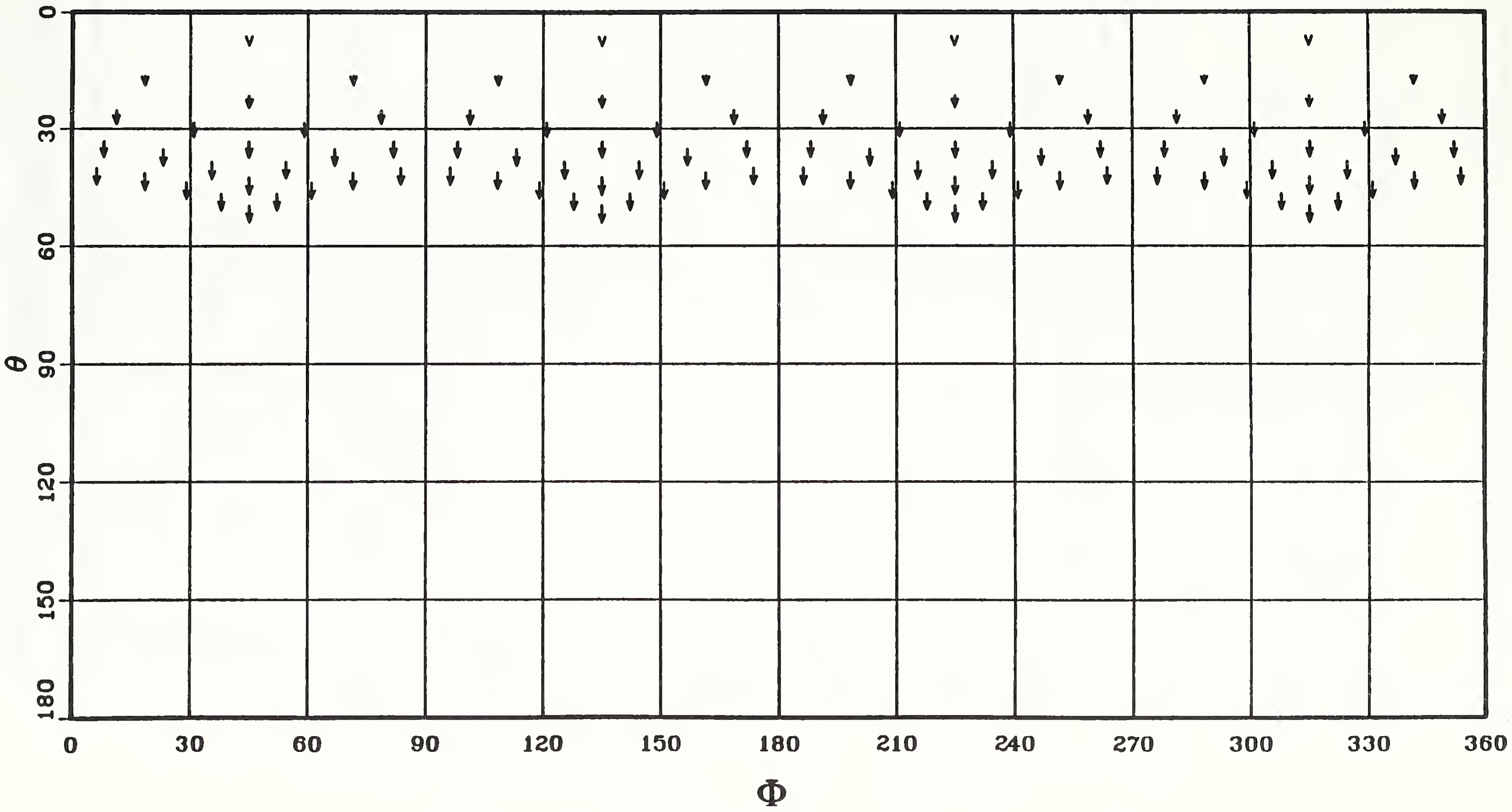


Figure 7d: ISOMETRIC REPRESENTATION OF OPTICAL FLOW

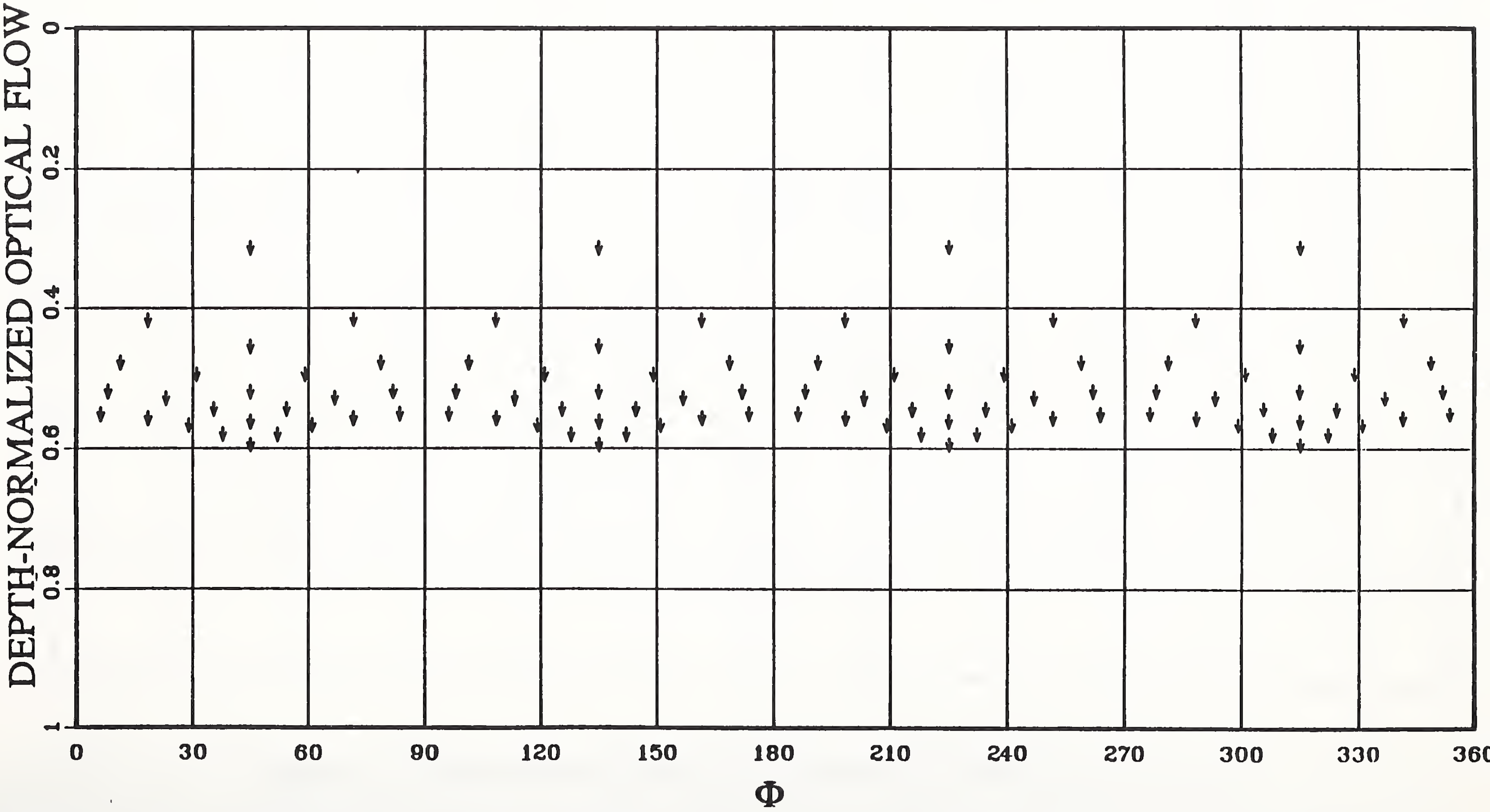
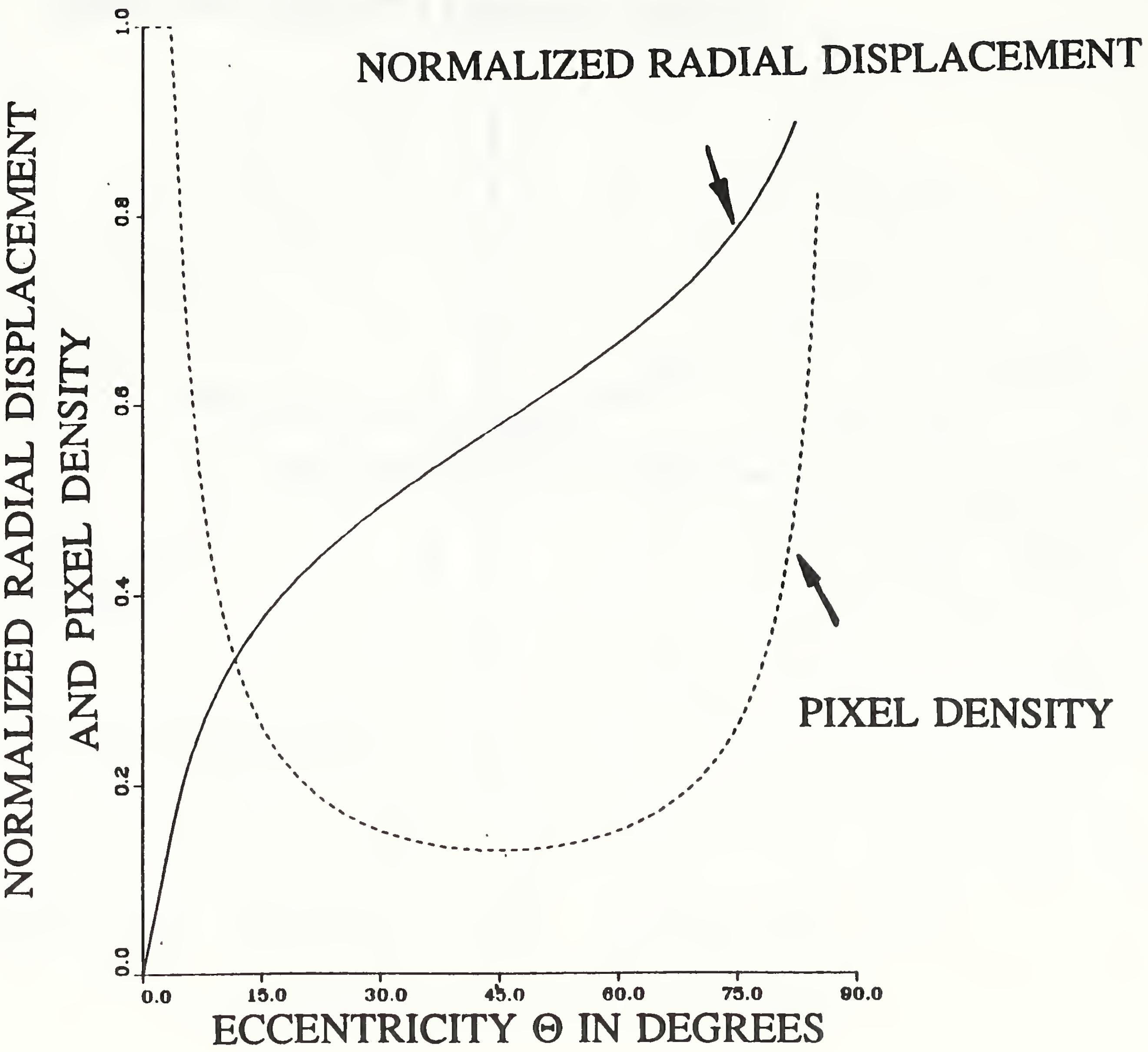


Figure 7e: ISOMETRIC REPRESENTATION OF DEPTH- NORMALIZED OPTICAL FLOW

Figure 7f: LOG TAN Θ RETINA (DEPTH)



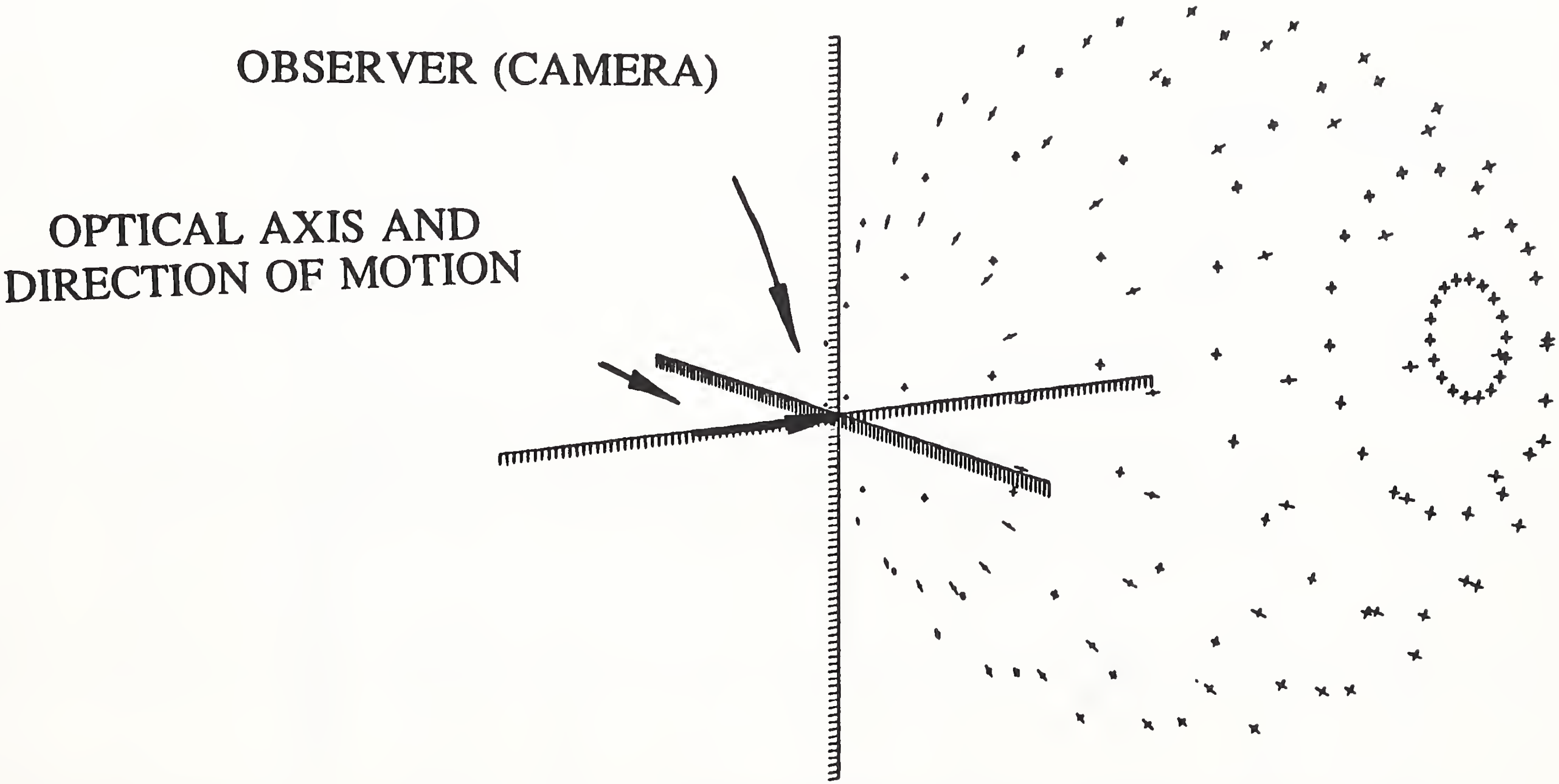


Figure 8a: EQUAL LOOMING POINTS

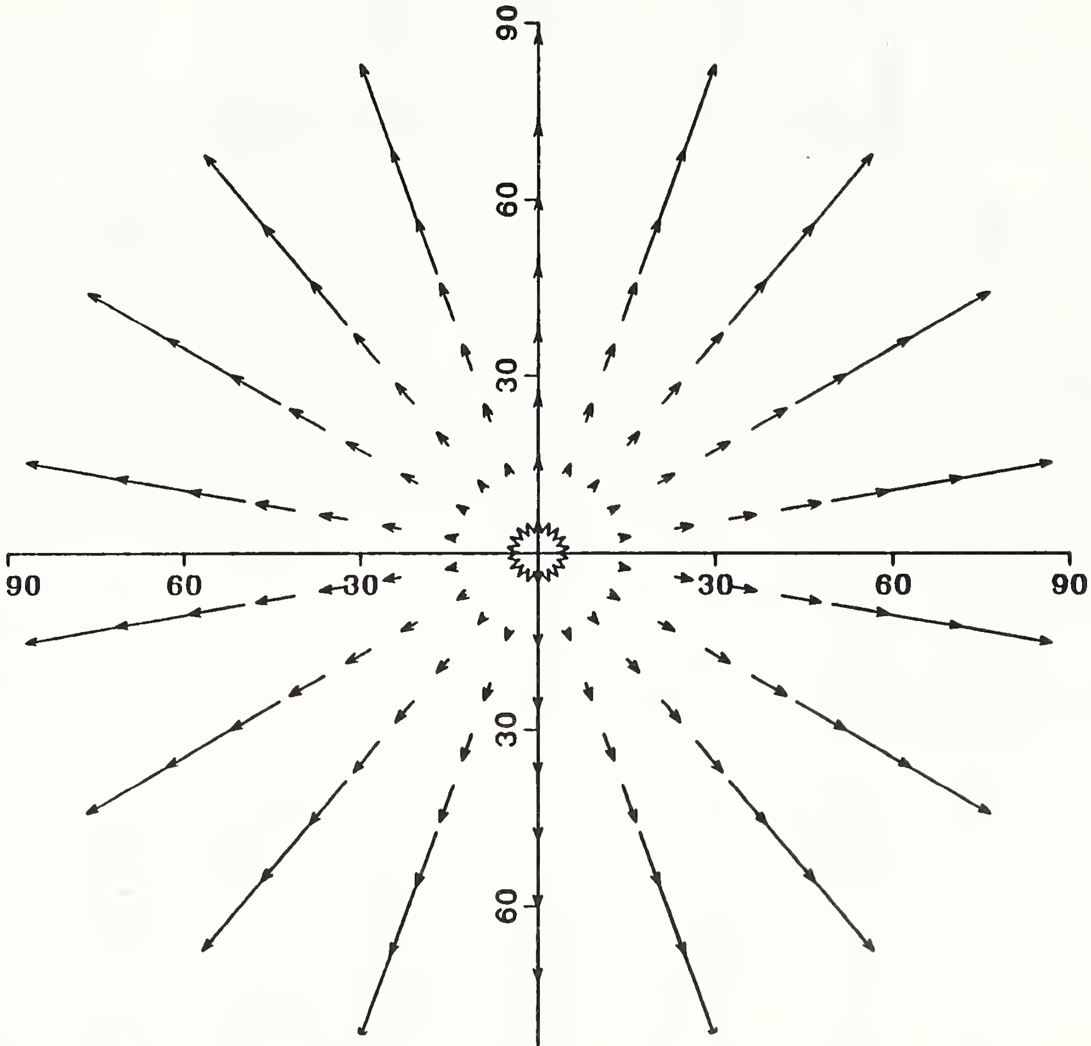


Figure 8b: OPTICAL FLOW GENERATED BY EQUAL-LOOMING POINTS

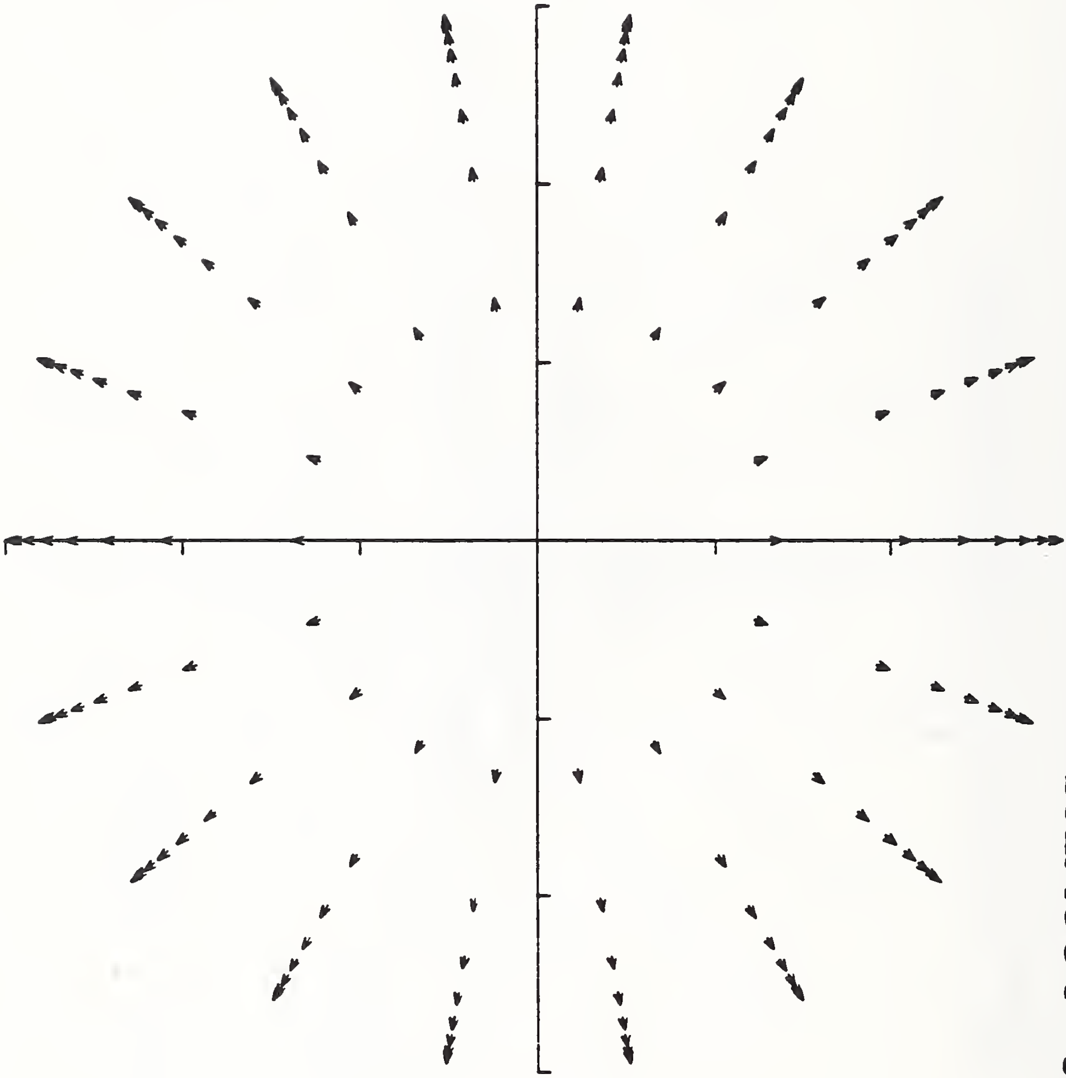


Figure 8c: LOOMING NORMALIZATION OF OPTICAL FLOW

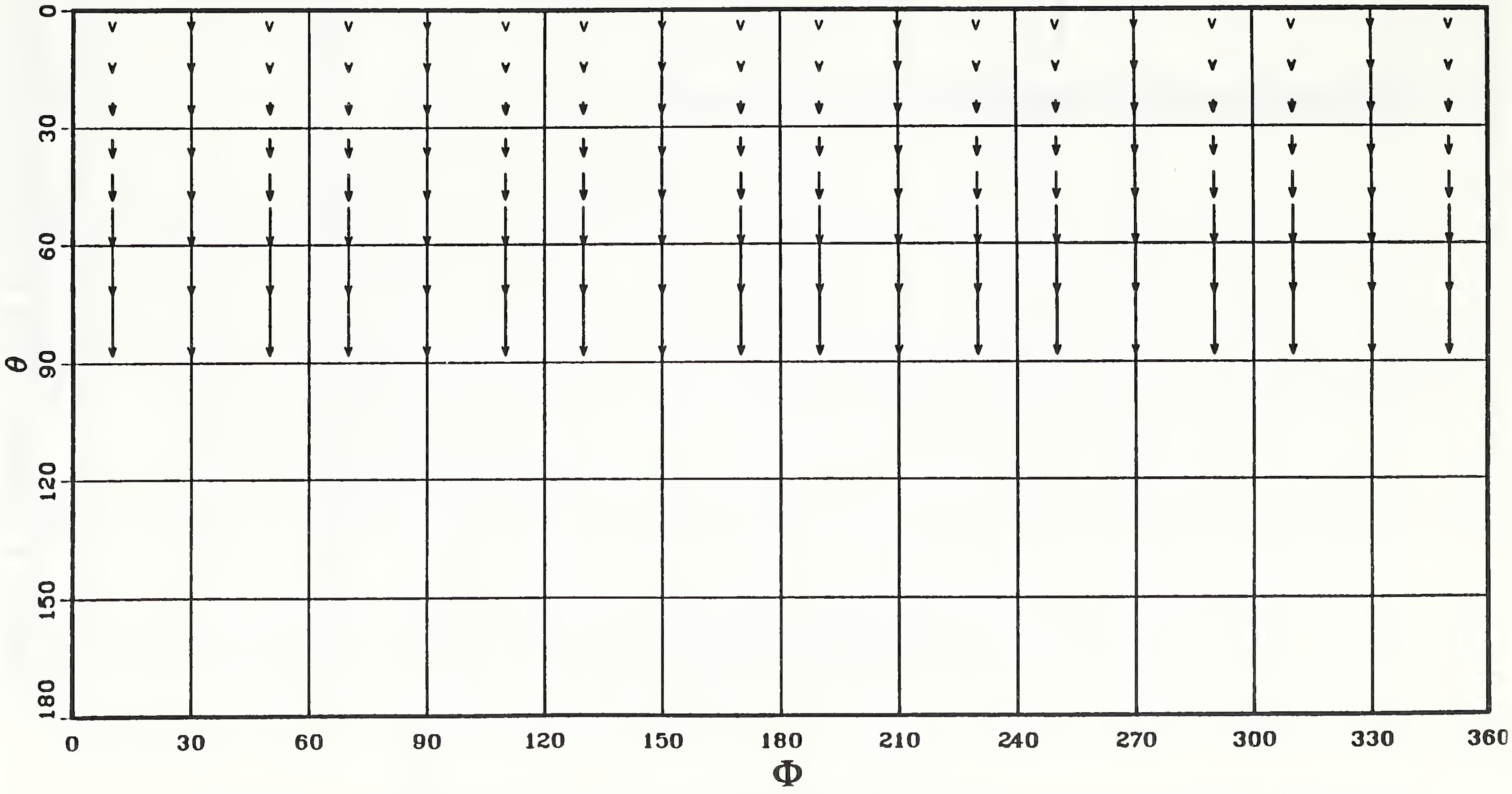


Figure 8d: ISOMETRIC REPRESENTATION OF OPTICAL FLOW

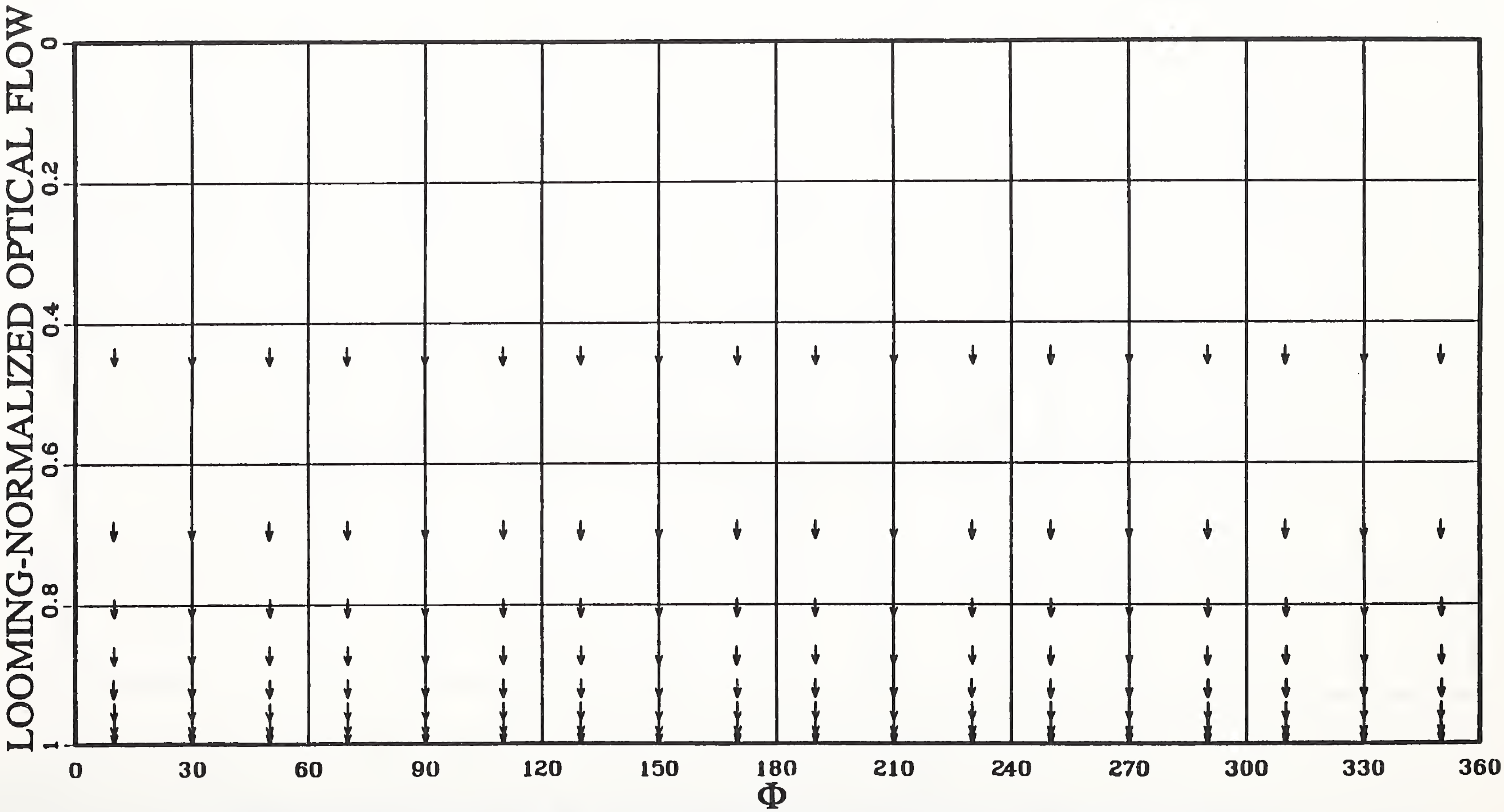
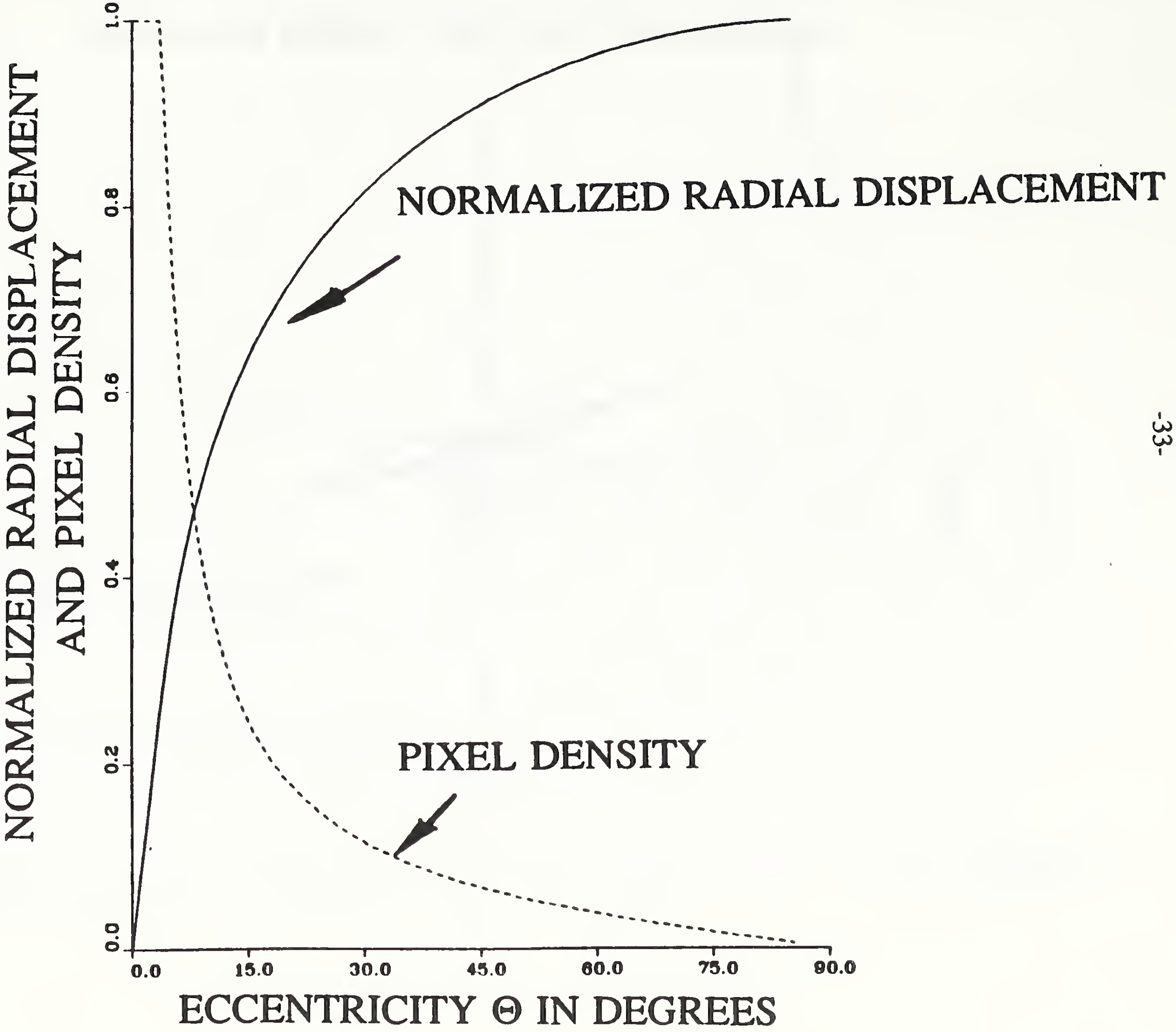


Figure 8e: ISOMETRIC REPRESENTATION OF LOOMING-NORMALIZED OPTICAL FLOW

Figure 8f: LOG SIN Θ RETINA (LOOMING)



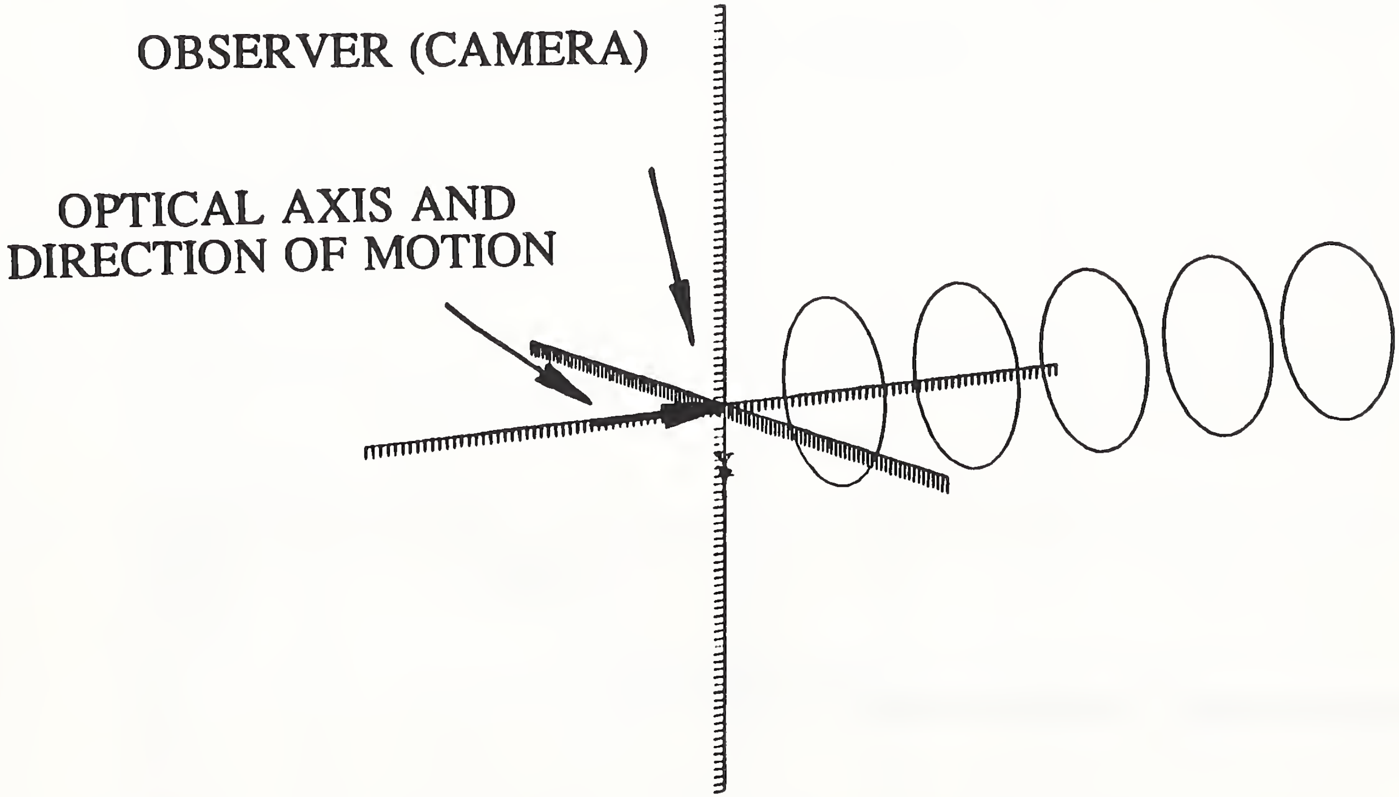


Figure 9a: EQUAL CLEARANCE POINTS

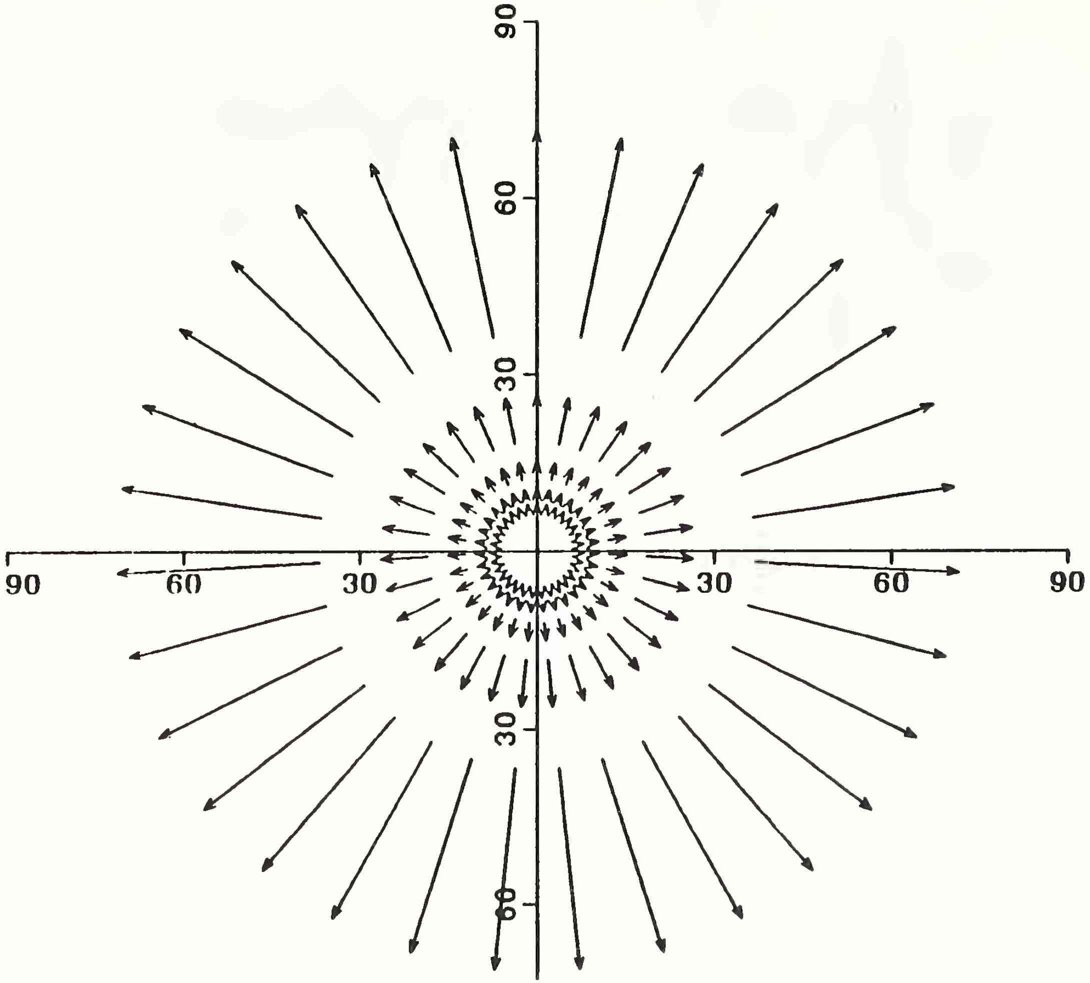


Figure 9b: OPTICAL FLOW GENERATED BY POINTS ON A CYLINDER

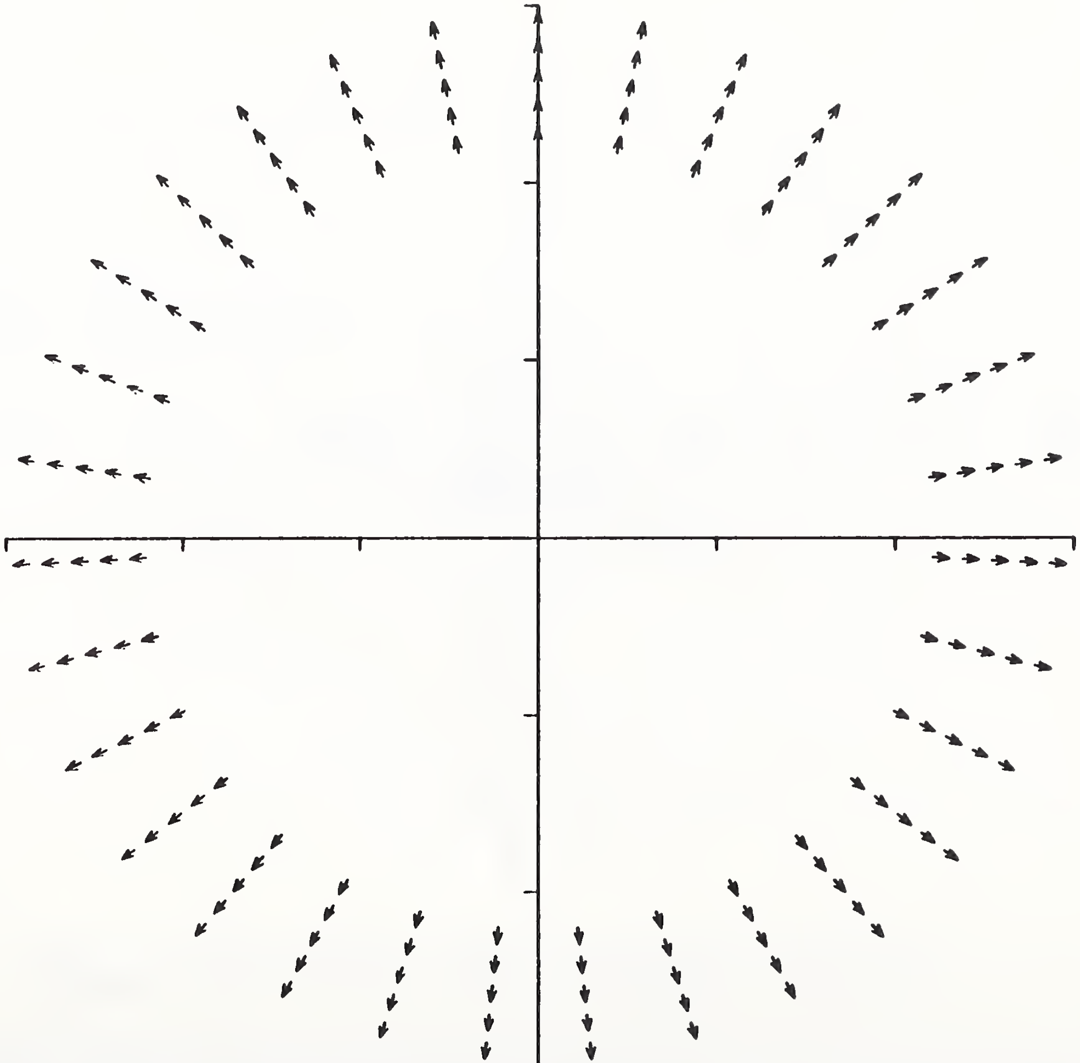


Figure 9c: CLEARANCE NORMALIZATION OF OPTICAL FLOW

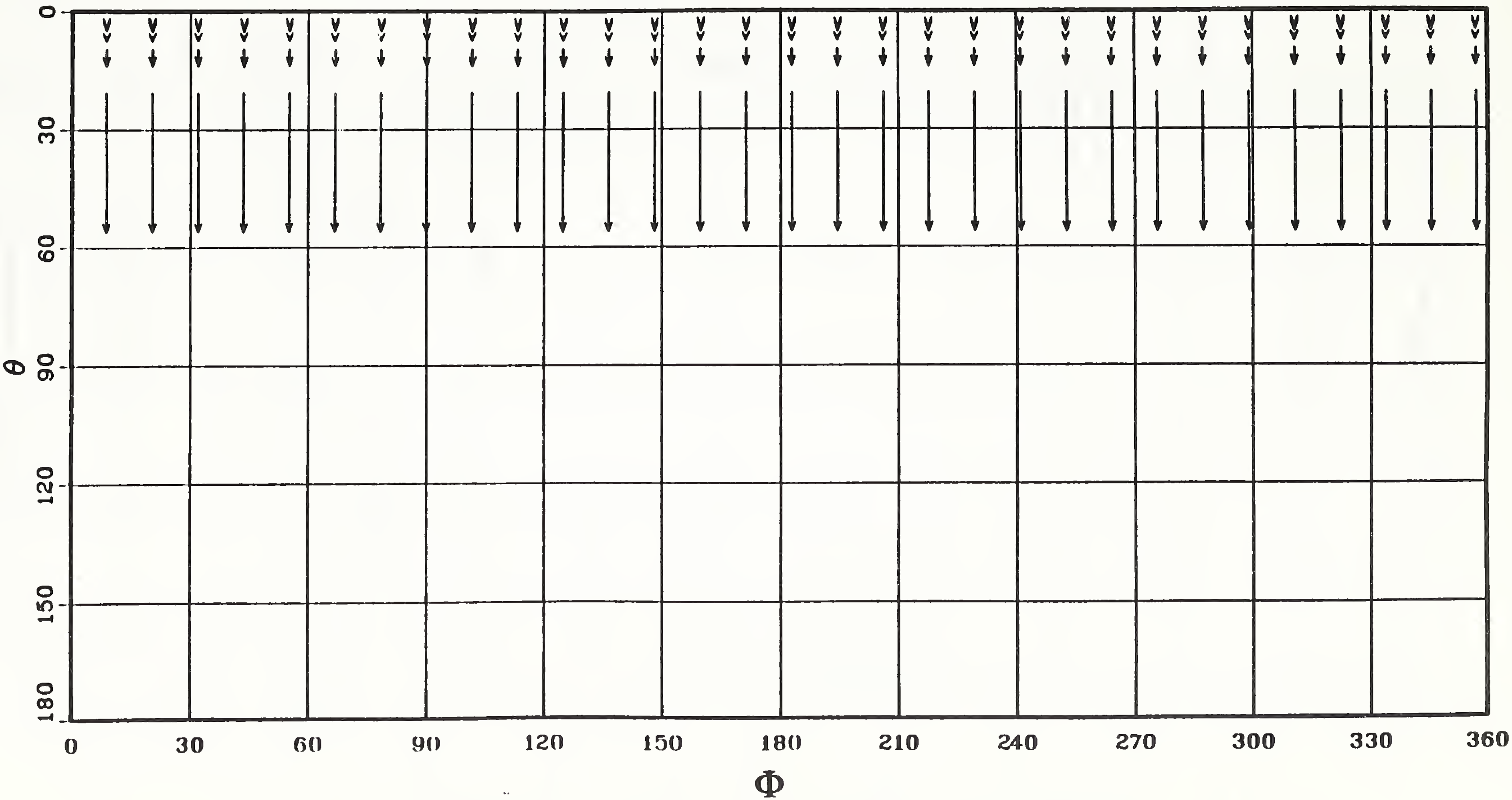


Figure 9d: ISOMETRIC REPRESENTATION OF OPTICAL FLOW

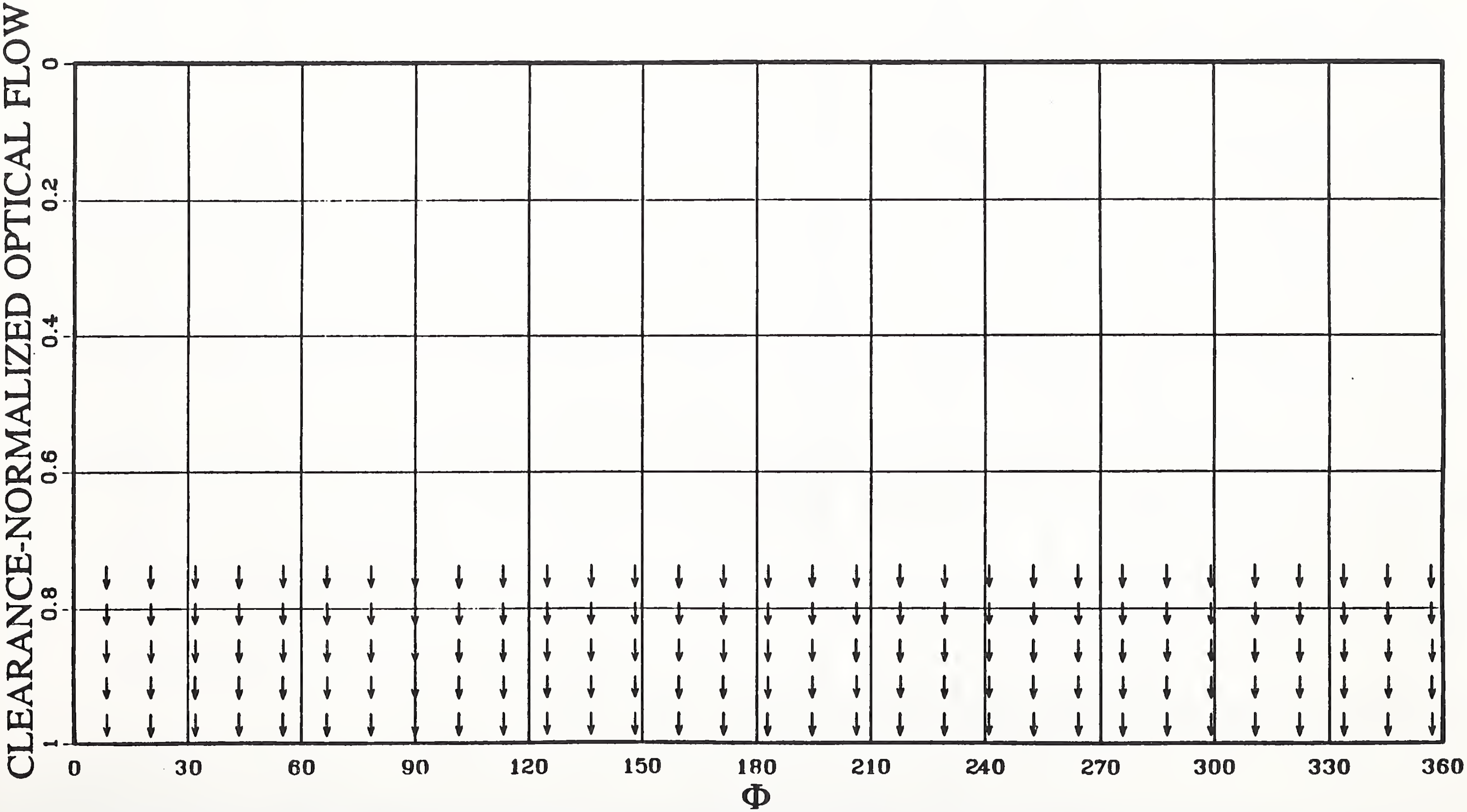
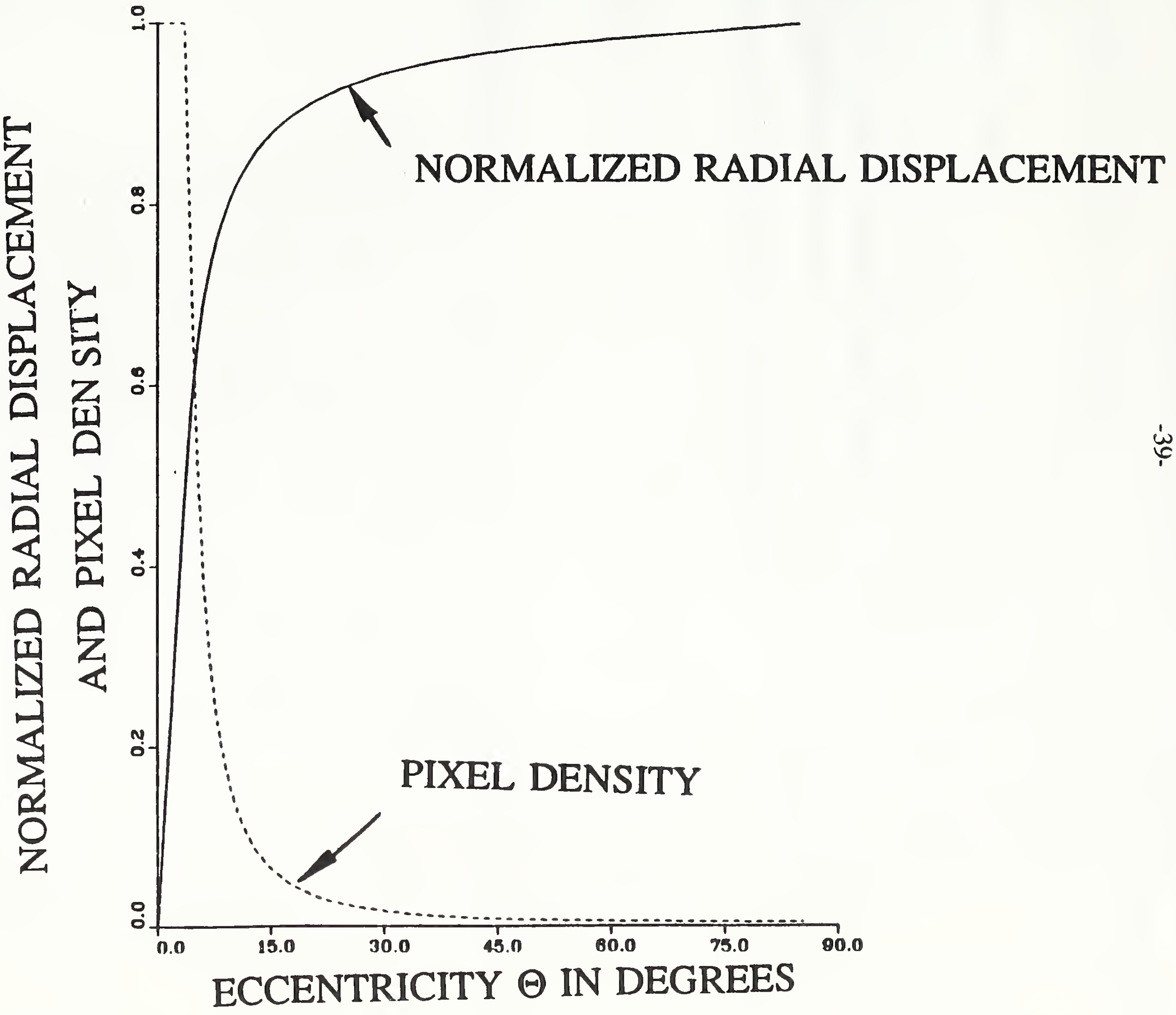


Figure 9e: ISOMETRIC REPRESENTATION OF CLEARANCE-NORMALIZED OPTICAL FLOW

Figure 9f: COT Θ RETINA (CLEARANCE)



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. ABSTRACT (A 200-WORD OR LESS FACTUAL SUMMARY OF MOST SIGNIFICANT INFORMATION. IF DOCUMENT INCLUDES A SIGNIFICANT BIBLIOGRAPHY OR LITERATURE SURVEY, MENTION IT HERE.)

This paper suggests several iconic image ''warpings'', or remappings, which facilitate computationally inexpensive measurements of moving 3-D points relative to a camera. Assuming translational motion of the camera, where the optical axis coincides with the direction of motion, and a stationary scene, points in 3-D space that lie on a particular 3-D surface produce a constant value for some nonlinear function of the optical flow. This function need not be computed after the image is formed, but rather can be implemented by hardware at the retinal level, i.e., via non-linear variable-resolution (usually logarithmic) retina. Four sets of different surfaces are introduced and there is one optical-flow-based constant value for each surface. We call these values "invariants". An invariant, which is a scalar, describes a 3-D surface. For each invariant a logarithmic retina is defined which will cause otical flow on these surfaces to have identical values.

The process of image remapping, called ''normalization'', is defined for four 1-D parameterizations of space: range, depth, looming and clearance. For each invariant a camera-retina imaging model utilizing spherical projection and foveal peripheral resolution is described for analyzing optical flow. Computer simulation demonstrates how the new suggested retinae normalize the optical flow with respect to each one of the parameterizations.

2. KEY WORDS (6 TO 12 ENTRIES; ALPHABETICAL ORDER; CAPITALIZE ONLY PROPER NAMES; AND SEPARATE KEY WORDS BY SEMICOLONS)

Visual motion; Logarithmic Retinae; Computer Vision

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